

Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 50 minutes

Please read the questions carefully before answering

1. (20 points) State the converse, contrapositive and negative of the following statement about functions taking real numbers to real numbers :

If the function $f(x)$ is continuous in $[a, b]$ then it is integrable in $[a, b]$.

2. (20 points) Use truth table to check that $p \wedge \neg q$ and $(\neg p) \vee q$ are negatives of each other.

3. (20 points) Prove the following statements or give counterexample if false:

a) If x, y are irrational real numbers, then $x + y$ is irrational.

b) Every continuous real valued function on real numbers function has a maximum (peak) or minimum (bottom) on the real number line.

4. (20 points) Prove using the contrapositive or contradiction: You must first state the contrapositive.

1. If $x + y + z > 30$ then one of x, y, z is greater than 10.

2. If n does not have a divisor $d_1 \geq \sqrt{n}$ (other than n itself) then it does not have a divisor $d \leq \sqrt{n}$ (other than 1).

5. (20 points) Prove that the composition of two surjective functions is surjective :

$$f : A \rightarrow B \text{ is onto}, g : B \rightarrow C \text{ is onto} \implies g \circ f : A \rightarrow C \text{ is onto}.$$

6. (Extra credit 10 points) Prove using contradiction that cube root of 4, denoted $\sqrt[3]{4}$, is irrational.

You may need the following fact about integers: a prime number p divides ab , i.e $p|ab$ means $p|a$ or $p|b$.