

PROOFS NOTES 9/28/26

Equivalence relations

Exercise 6.5:

Which of reflexive, symmetric and transitive properties are satisfied by each of following?

a) Set: Natural numbers. Relation: xRy iff x divides y

Reflexive : Yes. x divides x for any x .

Symmetric: No. 2 divides 4 but not vice versa.

Transitive: Yes. If x divides y , and y divides z , then x will divide z .

More precisely: If $y = xk$, $z = ym$, then $z = x(km)$.

b) Set: Sets. Relation: ARB iff A is a subset of B .

Reflexive: Yes. Any set is a subset of itself.

Symmetric: No. $\{1\}$ is subset of $\{1,2\}$ but not vice versa.

Transitive: Yes. Similar to (a)

c) Set: People in a school. Relation: xRy iff x likes y .

Reflexive: Yes, everybody likes themselves.

Symmetric: No.

Transitive: No.

d) Set: $\{1,2,3\}$. Relation: $\{(1,1), (1,2), (2,2), (1,3), (3,3)\}$ (1 is related 1, 1 is related to 2, etc)

Reflexive: Yes. $(1,1)$, $(2,2)$ and $(3,3)$ are present.

Symmetric: No. $(1,2)$ is present but $(2,1)$ is not.

Transitive: Yes. Whenever (a,b) and (b,c) are there, (a,c) is also there. Note that you don't need any two elements to be related. $(1,2)$ and $(1,3)$ are there but $(2,3)$ is not, but it doesn't matter. It only matters that, IF (a,b) and (b,c) are there, (a,c) is also there.

e) Set: Real numbers. Relation: xRy iff $x-y$ is rational.

Reflexive: Yes. $x-x = 0$ is rational for any x .

Symmetric: Yes, If $x-y$ is rational, then $y-x$ is also rational because $y-x = -(x-y)$.

Transitive: Yes, similar to xRy if $x-y = 5m$

If $x-y$ is rational and $y-z$ is rational then $x-z = (x-y)+(y-z)$ and rational plus rational is rational so $x-z$ (and hence $z-x$) is rational. So xRy and yRz means xRz .

f) Set: Real numbers. Relation: xRy iff $x-y$ is irrational.

Reflexive: No. $x-x = 0$ is rational.

Symmetric: Yes. Same kind of proof as in (e).

Transitive: No. Irrational plus Irrational could be rational.

g) Set: Real numbers. Relation: xRy iff $(x-y)^2 < 0$.

Empty relation. No two real numbers are related. $(x-y)^2 > 0$ always.

Not reflexive because $(x-x)^2 = 0$ and not < 0 .

Symmetric and transitive. (Vacuously, because there are no relations to start with, similar to how (d) was transitive).

h) Set: Real numbers. Relation: xRy iff $|x-y|$ less than or equal to 2.

Reflexive: Yes. $|x-x| = 0$ is not greater than 2.

Symmetric: Yes. Because $|x-y| = |y-x|$

Transitive: No. Example: $1R3$ and $3R5$ but 1 is not related to 5.