3/26/2025 Spring 2025, Patterns in Environmental Mathematics
Test II solutions Sitaraman

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1. (10 points) How do climate scientists know how much carbon there was over the past thousands of years?

   Solution:

   Carbon from the CO2 coming from burning fossil fuels has different carbon isotope composition than the carbon in the atmosphere.

   We can find the carbon isotope composition at different times by analyzing tree rings or the air trapped in ice cores from Antarctica.

   Since 1850 the isotope composition has changed dramatically showing that more carbon from fossil fuel burning has entered the atmosphere, thus altering its carbon isotope composition.

2. (15 points) 1000 dollars is invested in a bank account at *the beginning of every year* at an interest rate of 10 per cent per year, for 3 years. Find the total amount in the bank after 3 years if interest is compounded annually, assuming the total amount is withdrawn at the beginning of the third year, after the last deposit is made. You will need to use the formula for the sum of a geometric sequence. How much was the net earnings? (i.e, total amount minus total deposit). You may need that $1.1^3 = 1.331$.

   Solution:

   We have $r = 0.1$ (divide 10 by 100 to convert from percentage to fraction).

   Note that this r is different from the common ratio of a geometric sequence. Also $P = 1000$.

   Here t is different for the amounts deposited in different years.

   The first 1000 collects interest for 2 years (as it is withdrawn *at the beginning of third year*, just for the sake of argument), the second 1000 collects interest for 1 year and the last 1000 is withdrawn right away (yes, such things can happen!), so it collects no interest.

   As discussed in class total amount is given by

   $$1000(1.1)^2 + 1000(1.1)^1 + 1000.$$

   Now we use the formula for the sum of a geometric sequence

   $$A = a\left(\frac{1-R^n}{1-R}\right)$$

   Here R is common ratio and n is number of terms.

   Plugging in $R = 1.1$ and $n = 3$ we get

   $$A = 1000\left(\frac{1-1.1^3}{1-1.1}\right) = 1000(-0.331/(-0.1)) = \$\ 3310.00.$$

   Now total deposit is 3000 dollars.

   Net earnings is $3310.00 - 3000 = \$\ 310.00$.

3. (15 points) Solve the following using the properties of logarithms. You may leave final answer in terms of logarithms and square-roots.

   (a) $\ln(x-1)+\ln(x+1)=0$.

   (b) $0.5^x = 0.01$.

   (c) $x = e^{\ln(0.0005)}$.

   Solution:

   (a) $\ln(x-1)+\ln(x+1)=0 \implies \ln((x-1)(x+1))=0 \implies x^2-1=1$.

   Here $RHS = 1$ because $\ln A = 0$ means $A = e^0 = 1$.

   In general, logarithm of 1 is zero no matter what base you use.

   So $x^2 = 2$ and $x = \sqrt{2}$.

   Note that $x = -\sqrt{2}$ is not valid because it will give negative values for $\ln(x-1)$ and $\ln(x+1)$ and logarithm is not defined for negative values.

   (b) $0.5^x = 0.01 \implies x\ln(0.5) = \ln(0.01) \implies x = \ln(0.01)/\ln(0.5)$

   (c) $x = e^{\ln(0.0005)} \implies x = 0.0005$ because ln function is inverse of exponential function, and they "cancel" each other out.

   The last 3 problems involve the following data:

   These are the prices of solar panel cells from 2001 to 2010:

   2.5,2.1,1.9,1.9,2.2,2.0,2.2,1.9,1.2,1.1

4. (15 points) Make a stem-leaf, tally and frequency table. Your table will have two rows, one for prices of \$ 2 and above, another for prices of \$ 1 and above.

   Solution:

   1 $0.9, 0.9, 0.9, 0.2, 0.1$ $\not{|||\,|||}$ 5
   2 $0.5, 0.1, 0.2, 0.0, 0.2$ $\not{|||\,|||}$ 5

5. (15 points) Find the following:

   (a) average (b) the standard deviation of the prices (You can leave your answer as a square root). (c) the price range for 95 percent of the data (assuming it is normally distributed).

   Solution:

   (a) There are 10 data points. So to find the average μ we need to add them and divide by 10.

$$\mu = \frac{2.5+2.1+1.9+1.9+2.2+2.0+2.2+1.9+1.2+1.1}{10} = 1.9.$$

   (b)The standard deviation σ is given by

$$\sqrt{\frac{(2.5-1.9)^2+(2.1-1.9)^2+(1.9-1.9)^2+(1.9-1.9)^2+(2.2-1.9)^2+(2.0-1.9)^2+(2.2-1.9)^2+(1.9-1.9)^2+(1.3-1.9)^2+(1.1-1.9)^2}{10}}$$

$$= \sqrt{\frac{0.6^2+0.2^2+0^2+0^2+0.3^2+0.1^2+0.3^2+0^2+0.6^2+0.8^2}{10}}$$

$$= \sqrt{\frac{0.36 + 0.04 + 0.09 + 0.01 + 0.09 + 0.36 + 0.64}{10}} = \sqrt{0.159} \simeq 0.4.$$

(c) Assuming normal distribution (bell curve), 95 percent of the prices would be within 2 standard deviations within the average. Two standard deviations equals $2\sigma = 2(0.4) = 0.8$. So 95 percent of the values would be between $1.9 - 0.8$ and $1.9 + 0.8$ or between 1.1 and 2.7. Actually in this case all the values lie in this range, so maybe this doesn't follow the normal distribution (bell curve).

6. Find the median, the mode, and the value in the 70th percentile.

   Arranging in ascending order, we get

   1.1, 1.2, 1.9, 1.9, 1.9, 2.0, 2.1, 2.2, 2.2, 2.5.

   The median is the average of the 5th and 6th data points, namely $(1.9 + 2.0)/2 = 1.95$.

   The mode is 1.9 since it appears 3 times, more than any other data point.

   The 70th percentile means 7 values are at or below that price, so it is the 7th value namely 2.1.