

Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 50 minutes

Please read the questions carefully before answering

1. (20 points) Prove using induction that $2^{n+1} < 1 + (n + 1)2^n$ for all $n \in \mathbb{N}$.
2. (Base 3 expansion) Prove using strong induction that every positive integer n is either a power of 3 or can be written as the sum of powers of 3 with coefficients 1, 2, or 0. In other words, $n = b_0 + b_13 + b_23^2 + \dots + b_k3^k$ for some positive integer k and the b_i are 1, 2, or 0.
3. (20 points) Prove using basic definition of limit that if s_n, t_n are convergent and $\lim_{n \rightarrow \infty} s_n = s, \lim_{n \rightarrow \infty} t_n = t$, then $\lim_{n \rightarrow \infty} (s_n - t_n) = s - t$.
4. (20 points) Prove that the recurrence sequence given by $s_1 = 1, s_n = \sqrt{4s_{n-1} + 1}$ is monotonic increasing, and bounded. Without finding the limit, explain why the limit should exist.

5. Check if following are true. To disprove something, enough to give ONE counterexample. But to establish it to be true, need to provide a proof. Examples are not enough. Each 5 points.
- a) The limit of the quotient of any two sequences will equal the quotient of the limits.
 - (b) For a function to be continuous at $x = c$ it is enough if the function is defined at $x \rightarrow c$.
 - (c) A bounded, increasing sequence of irrational numbers always has a irrational number as least upper bound.
 - (d) For any sequence of real numbers $x_n \rightarrow c$, with c being any real number, $\tan x_n \rightarrow \tan c$.
6. (Challenge, extra credit 20 points) Prove using the definition of limits that every real number is a limit of a sequence of rational numbers of the form $a_i/10^{k_i}$ where a_i is an integer and k_i is a non-negative integer. In other words, decimal expansion exists for all real numbers. (You cannot start with a decimal expansion! Goal is to prove it exists).