Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (15 points) Prove directly that A is a subset of C iff $A \cap C$ equals A.

   Start by stating the condition for any two sets to be equal. Then use that condition in your proof of above statement.

2. (a) (10 points) Show that $aRb \iff a - b \in \mathbb{Q}$ (the rational numbers) is an equivalence relation on the real numbers $\mathbb{R}$. Note that $a, b \in \mathbb{R}$.

   (b) (5 points) Find equivalence class of 0.

3. (15 points) Let X be the set of real numbers on the real number line.

   Show that $aRb \iff a \geq b$ is NOT an equivalence relation.

   What conditions are satisfied and what conditions are not satisfied?

4. (10 points) Prove by cases: if 2 divides $x - y$ then 2 divides $x + y$.

5. (15 points) Prove using contradiction: $\sqrt[3]{3}$ (Cube root of 3) is irrational.

6. (15 points) Prove by contradiction that there cannot be a bijection between a finite set and an infinite set.

7. Check if following are true. To disprove something, enough to give ONE counterexample. But to establish it to be true, need to provide a proof. Examples are not enough. You can quote a theorem as part of the proof but saying "this was proved in class" is not enough.

   a) (5 points) The set of all real numbers with terminating decimal expansion is a countable set.

   (b) (5 points) The union of the equivalence classes of a set under any equivalence relation equals the whole set.

   (c) (5 points) The set of functions from a finite set to another finite set is countable.

8. (Challenge problem, extra credit 20 points) This exercise shows that there cannot be a bijection between a set S and its power set $P = \mathcal{P}(S)$, namely the set of all subsets of S. So for example the power set of $\mathbb{N}$ (natural numbers 1,2,3,.....) is uncountable.

   It is enough to show there no surjection $S \to P$. Suppose f is a surjection.

   Question : Show that the following subset cannot be in $f(S)$:

$$T = \{x \in S \ : \ x \notin f(x)\}.$$