12/9/2025 Final Exam Fall 2025,Proofs and Problem Solving Sitaraman

Howard University Math Department

Instructions: **NO CALCULATORS OR CELLPHONES**

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 120 minutes; Total 100 points.

Please read the questions carefully before answering.

1. (10 points) Prove by cases: For any two real numbers x, y, $max\{x, y\} = \frac{x + y + |x - y|}{2}$.

2. (20 points) Write the negative, converse and contrapositive of the following statement. Then prove it if it is true, give a counterexample if it is false: "If a function maps the finite set X ONTO the finite set Y, then every element in Y has a unique pre-image in X."

   (every element in Y has unique preimage is same as saying different elements in X have different images).

3. (10 points) Prove if true or give counterexample:

   For all sets A, B, and C: Either $C \subseteq A$ or $C \subseteq B$ if and only if $C \subseteq A \cup B$.

   Use the following definition: Given any two sets S, T, $S \subseteq T$ means that each element of S is also in T.

4. Let R be an equivalence relation on a set A. Let B be the set of equivalence classes of A. Define a function f from A to B by the rule $f(x) = [x]$.

   (a) (6 points) Show that this is an onto function.

   (b) (8 points) Given an example of a set A with an equivalence relation R such that f is a one-one function.

   (c) (6 points) Show that $f(x) = f(y) \iff xRy$.

5. (20 points) Prove by induction for all natural numbers n and a fixed real number x:

   If $2 + x > 0$ then $(2 + x)^n \geq 1 + n + nx$.

6. (10 points) Find a bijection (1-1, onto map) from the set of integers $\mathbb{Z}$ to the set of natural numbers $\mathbb{N} = \{1, 2, 3, ...\}$.

7. (10 points) Using basic definition of limit, show that $\frac{n+1}{n} \to 1$ as $n \to \infty$.

8. (extra credit 15 points) Prove by contradiction: (You must use basic definitions of limit and continuity, using ϵ, δ etc).

   If a continuous real valued function f has $f(c) > 0$ for $c > 0$ then for some $\delta > 0$ we have $f(x) > 0$ for all x such that $|x - c| < \delta$.