

Howard University Math Department

Instructions: **NO CALCULATORS OR CELLPHONES**

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 120 minutes; Total 100 points.

Please read the questions carefully before answering.

1. (10 points) Prove by cases: For any natural number n we have $n^3 + n$ is even.
2. (20 points) Write the negative, converse and contrapositive of the following statement. Then prove it if it is true, give a counterexample if it is false: "If a function maps the finite set X 1-1 to the finite set Y , then every element in Y has a pre-image in X ."
(every element in Y has preimage is same as saying function is onto).
3. (10 points) Prove if true or give counterexample:
For all sets A, B , and C :
 $C \subseteq A \cup B$ iff either $C \subseteq A$ or $C \subseteq B$.
Use the following definition: Given any two sets S, T , $S \subseteq T$ means that each element of S is also in T .
4. Let R be an equivalence relation on a set A . Let B be the set of equivalence classes of A . Define a function f from A to B by the rule $f(x) = [x]$.
 - (a) (6 points) Show that this is an onto function.
 - (b) (8 points) For the equivalence relation on \mathbb{N} given by xRy if $x + y$ is even find $f(1)$ and $f(2)$. You should describe the elements in each image.
 - (c) (6 points) Show that $f(x) = f(y) \iff xRy$.
5. (20 points) Prove by induction for all natural numbers n and a fixed real number x :
If $3 + x > 0$ then $(3 + x)^n \geq 1 + 2n + nx$.
6. (10 points) Find a bijection (1-1, onto map) from the the set of natural numbers $\mathbb{N} = \{1, 2, 3, \dots\}$ to the set of integers \mathbb{Z} .
7. (10 points) Using basic definition of limit, show that $\frac{n+2}{n} \rightarrow 1$ as $n \rightarrow \infty$.
8. (extra credit 15 points) Prove by contradiction: (You must use basic definitions of limit and continuity, using ϵ, δ etc).
If a continuous real valued function f has $f(c) > 0$ for $c > 0$ then for some $\delta > 0$ we have $f(x) > 0$ for all x such that $|x - c| < \delta$.