$$1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

$n=100 \quad 1+2+3+\cdots+100 = \frac{100(101)}{2} = 5050$

Proof by induction: $n=1$: $1 = \frac{1(1+1)}{2} = 1$ ✓

Assume true for k.

$$\underbrace{1+2+\cdots+k}+k+1 = \frac{k(k+1)}{2}+(k+1)$$

$$= (k+1)\left(\frac{k}{2}+1\right) = \frac{(k+1)(k+2)}{2}$$

Put $k+1$ in formula $\frac{n(n+1)}{2}$ to get $\frac{(k+1)(k+1+1)}{2}$
RHS

They are equal!

$$1^2+2^2+3^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$$

$n=1 \quad 1^2 = \frac{1(2)(3)}{6} = 1$ ✓

(Just checking: $n=2 \quad 1^2+2^2=5 \quad$ RHS: $\frac{2\times3\times5}{6}=5$
$n=3 \quad 1^2+2^2+3^2=14 \quad$ RHS: $\frac{3\times4\times7}{6}=14$)

Assuming true for $n=k$, prove for $n=k+1$

$$\underbrace{1^2+2^2+3^2+\cdots+k^2}_{\shortparallel}+(k+1)^2 \stackrel{?}{=} \frac{(k+1)(k+2)(2(k+1)+1)}{6}$$

Use formula for k

$$\frac{k(k+1)(2k+1)}{6}+(k+1)^2$$

(what you get by plugging $k+1$ in RHS

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

Are the two sides equal?

(If you just put $n=k+1$ in formula, would it equal what you get by adding $(k+1)^2$ to the sum of first k squares?)

$$LHS = \frac{k(k+1)(2k+1)}{6} + (k+1)^2$$

$$= (k+1)\left[\frac{k(2k+1)}{6} + k+1\right]$$

$$= (k+1)\left[\frac{2k^2+7k+6}{6}\right]$$

$$RHS = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= (k+1)\frac{(2k^2+7k+6)}{6}$$

Yes, they are equal

Formula works for $k+1$ if it works for k