MATH 103-1 Spring 2025
Final Exam A
4/30/2025
PLEASE DO NOT SHARE THIS EXAM WITH ANYONE
THANK YOU.
PLEASE BE RIGOROUS AND CONCISE IN WRITING.

1. [15*pts*] There are 140 Math Majors in the department of Mathematics.
Of these, 52 are taking Biology, 71 Chemistry and 40 Physics.
Of these Maths Majors, 15 are taking both Biology and Chemistry, 8 are taking are taking Chemistry and Physics, 11 are taking Biology and Physics, and 2 are taking all 3 courses.
(i) How many students are not taking any of these 3 courses?
(ii) Draw a Venn Diagram to illustrate the enrollments of these Maths Majors in the 3 courses

2. [10*pts*] Let $f : A \to B$ be a function and $C \subseteq A$.
Prove that if $f$ is injective then $f^{-1}(f(C)) = C$

3. [15*pts*] Prove by induction that $P(n)$ is true for all $n \geq 1$
$P(n) : 2 + 6 + 10 + ... + (4n + 2) = 2n^2$

4. [10*pts*] Let $f(x) = x^2 + 2$. Show using the definition of limit (with $\varepsilon - \delta$) that $\lim_{x \to 1} f(x) = 3$. Is $f$ continuous at 1?

5. [10*pts*] Using the definition of a derivative, show that if $f(x) = x^{1/3}$ then $f'(x) = \frac{1}{3}x^{-2/3}$.

6. [15*pts*] Define a relaion $R$ on $\mathbb{Z}$ by:
$a, b \in \mathbb{Z},\ aRb \iff a \equiv b \pmod 5$
(i) Show that $R$ is an equivalence relation on $\mathbb{Z}$
(ii) Describe $E_7$ the equivalence class of 7
(iii) How many distinct equivalence classes are there?

7. [10*pts*] Prove that for every $n \in \mathbb{Z}$, $\quad n^2 + 7n + 6$ is always even

8. [15*pts*] The function $f(x) = x^3 - 3x + 1$ is continous and differentiable on $\mathbb{R}$
(i) Explain why $f$ satisfies the hypotheses of the Mean Value Theorem on $[-2, 2]$
(ii) Find all value(s) $c \in (-2, 2)$ at which the conclusion of the MVT hold.