

## PROOFS AND PROBLEM SOLVING

10-27-2025

10.9 (Chapter 10) Prove using induction  $5^{2n} - 1$  is divisible by 8.

First step:  $n=1$  :  $5^2 - 1 = 24$  is divisible by 8.

Assume true for  $n$ .

Need to prove for  $n+1$  using result for  $n$ .

$$5^{2(n+1)} - 1 = 5^{2n+2} - 1 = (5^{2n}(25)) - 1 =$$

We know  $5^{2n} - 1 = 8m$  for some integer  $m$ . So  $5^{2n} = 8m + 1$  and this can be used in above equation.

$$((8m + 1)25) - 1 = 8m(25) + 25 - 1 = 8m(25) + 24 \text{ which is a multiple of 8.}$$