PROOFS AND PROBLEM SOLVING

10-21-2025

10.14 (Chapter 10) Bernoulli's Inequality: If $1+x > 0$ then $(1+x)^n \geq 1+nx$ for all natural numbers n.

For $n=1$ we have $1+x = 1+x$ so it works.

Assume true for n, and prove for n+1. In other words, prove $(1+x)^{n+1} \geq 1+(n+1)x$.

$(1+x)^{n+1} = (1+x)^n(1+x)$ (to use the induction hypothesis, namely $(1+x)^n \geq 1+nx$).

Because $1+x > 0$, we can say $(1+x)^n(1+x) \geq (1+nx)(1+x)$

Note that we are multiplying both sides by a positive number, so the inequality doesn't change direction.

Now if we prove that $(1+nx)(1+x) \geq 1+(n+1)x$ then we will be done.

But $(1+nx)(1+x) = 1+(n+1)x + nx^2 \geq 1+(n+1)x$ is true because nx^2 is always nonnegative.