9/16/2019 Fall 2019, Proof and Problem Solving I Quiz 2 Solutions Sitaraman

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1. (10 points) Write the negative and converse of the following:

   If the derivative is zero at a point then the function is either local maximum or local minimum there.

   Is the statement true? Is its converse true?

   Solution: Negative: If the derivative is zero at a point then the function is neither local maximum nor local minimum there.. $(A \implies NOT\ B)$

   Converse: If the function is either local maximum or local minimum at a point then the derivative is zero there.

   NOTE: opposite of the OR statement"A OR B" is "NOT A AND NOT B"

   So opposite of " either local maximum or local minimum" is "neither local maximum nor local minimum."

   Reason: If negative is true then statement must be false. In order for "A OR B" to be false both of them must be false.

   Is statement true? Is its converse true?

   Answer: the statement and its converse are both false.

   If you have an inflexion point then derivative is zero at that point but the function ineed not have local maximum or local minimum there.

   Another example is a constant function $y = k$. Derivative is always zero but it has no minima or maxima.

   Converse is also false! Absolute value function has a GLOBAL minimum at 0 but its derivative is not zero there.

   In fact, derivative doesn't exist!

   NOTE: By Fermat's theorem on extrema, IF DERIVATIVE EXISTS, then the converse is true!

   Prove the following by contradiction or contrapositive. If the statement is not true then prove that it is false using counterexample.

2. (10 points) $(A \cup B) \cap C = A \cup (B \cap C)$ for all sets A, B, C.

   Solution:

   This is false. Counterexample: $A = \{1, 2, 3\}, B = \{2, 4, 5\}, C = \{3, 6\}$.

   Then $(A \cup B) \cap C = \{1, 2, 3, 4, 5\} \cap C = \{3\}$

   But this is not equal to $A \cup (B \cap C) = A \cup \{\ \} = A = \{1, 2, 3\}$.

3. (10 points) if n^2 is odd then n is odd.

   Solution:

   Assume NOT B is true. Then n is even. So n^2 is also even because if 2 divides a number it divides its square as well! Proof: If $n = 2m$ then $n^2 = 4m^2 = 2(2m^2)$. This contradicts the assumption that n^2 is odd.