8/28/2019 Fall 2019, Proof and Problem Solving I Quiz 1 Solutions Sitaraman
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1. (10 points) What is wrong with the following statement?

   Football is a violent game. Football players make a lot of money. Therefore to make money we need to be violent.

   Solution: There are two things wrong. First is that it confuses correlation not being causation. There are many non-violent ways to make money. Secondly it seems to imply that football players are violent. Just because they play a violent game doesn't mean they are violent.

2. (10 points) Show that the statement "$f(n) = n^2+n+1$ is prime for all natural numbers" is false by giving a counterexample.

   [For $n = 1, f(1) = 3$, a prime. For $n = 2, f(2) = 2^2 + 2 + 1 = 7$, a prime. Is it true for the rest of the natural numbers?]

   Solution:

   For $n = 3, f(3) = 3^2 + 3 + 1 = 13$, a prime. For $n = 4, f(4) = 4^2 + 4 + 1 = 21$, not a prime.

   So the counterexample shows it is not true for all natural numbers, only some.

3. (10 points) It is known that $R(3, 4) = 9$. Show that it cannot be 6 by constructing an example of a graph that represents a group in which there is neither 3 who know each other NOR 4 who don't know each other.

   Solution:

   The hexagon will work.

   Clearly there is no triangle.

   Each person (point) knows two others (the adjacent points), so that leaves only 3 people not known to that person.

   Among those three people, clearly at least two know each other (in fact there are two pairs of adjacent points). This is true for each person, so it is not possible for there to be four people none of whom know each other. There can at most be three who don't know each other.