11/4/2026 Discrete Structures Midterm Test 2 Solutions Sitaraman

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1. (12 points) Check if the map $\mathbb{Z} \to \mathbb{Z}$ given by $f(m) = 2m + 1$ is 1-1 and onto. Prove your answer.

   (8 points) Find its inverse function if it exists. What is the domain of the inverse function?

   Solution:

   Proof:

   1-1 : $f(x) = f(y) \implies 2x + 1 = 2y + 1 \implies x = y$.

   ONTO: The image of any integer is an odd integer. So even integers have no pre-image. In other words, no integer maps to an even integer. So it is not onto.

   Inverse: Inverse exists because it is 1-1. Solving for x from $y = 2x + 1$ we get $x = (y - 1)/2$. So $f^{-1}(x) = (x - 1)/2$.

   This is defined because it is a 1-1 function. The domain of the inverse function, which is just the range of given function, is set of all odd integers.

2. (a) (12 points) Show that the following is an equivalence relation on $P(S)$, the set of subsets of $S = \{1, 2, 3, 4, 5, ..., 9, 10\}$. Note that the elements of $P(S)$ are the subsets, and NOT the numbers 1,2,3,....

   "Two subsets A and B are related iff they have same number of elements"

   (b) (8 points) Find the equivalence class of the empty set and the set $\{1\}$.

   (c) (5 points) Show that the equivalence classes are distinct : no two of them will share a common element.

   Easy to check that this is an equivalence relation.

   ARA for any subset A because it has same number of elements as itself.

   $ARB \iff BRA$ because A has same number of elements as B means B has same number of elements as A.

   Similarly ARB and BRC means ARC.

   The equivalence class of the empty set $\{\}$ is A such that it has the same number of elements as the empty set, which is zero. So it has only one element, namely the empty set.

   The equivalence class of the set $\{1\}$ is A such that it has the same number of elements as $\{1\}$ which is one. So it has ten elements, namely $\{1\}, \{2\}, ..., \{10\}$.

   Yes, equivalence classes will be distinct because two subsets in two different classes will have different number of elements, so there can be no common subset for different equivalence classes.

3. (20 points) State whether true or false. You must prove it if you think it is true. If it is false, provide a counterexample.

   (a) The sequence $n^2 - n + 1$ where $n \geq 1$ is increasing.

   (b) All equivalence classes under an equivalence relation will have the same number of elements.

   (d) If a relation is not symmetric then it is antisymmetric.

   (e) A partial order cannot be a total order.

   a) True. $n^2 - n + 1 < (n+1)^2 - (n+1) + 1$ because $n^2 - n + 1 < n^2 + 2n + 1 - n - 1 + 1$ because $0 < 2n$ if $n \geq 1$.

   b) False. For example, if you divide a set of students based on the relation that they have the same major, the students in same major form the equivalence classes. But different majors might have different number of students.

   c) False, the relation that says $xRy \iff x > y$ is neither symmetric nor antisymmetric.

   d) False, The relation $x \geq y$ gives a partial order as well as a total order on the set of real numbers.

4. (10 points) Find the GCD of 111 and 30 using Euclidean algorithm.

   Solution:

   $111 = 3(30) + 21$

   $30 = 1(21) + 9$

   $21 = 2(9) + 3$

   $9 = 3\ (3) + 0.$

   So 3 is the GCD.

5. For the following problems, you MUST explain the steps you took to arrive at final answer.

   (a) (6 points) How many outfits consisting of one pair of shorts, one shirt and a hat can you make from 5 shirts and 3 hats and 4 pairs of shorts ?

   (b) (7 points) How many choices do you have if you are packing 3 shirts for a trip from a set of 7 different shirts? (You are selecting 3 shirts).

   (c) (6 points) How many ways can you dress on 3 days with different shirts if you have a total of 7 shirts? For example, Mon - Shirt A, Tue – Shirt B, Wed – Shirt C and so on. Basically, you are arranging the 7 shirts in 3 places.

   (d) (6 points) If you have just 3 different shirts to wear for 7 days, but you can repeat the same shirt, how many possibilities are there for the whole week? For example, you can wear shirt A for all 7 days, or it can be something like AABBBCC.

Solution:

a) This is just $5 \times 3 \times 4 = 60$ because of multiplication principle. You are only picking one from each and the choices are independent.

(b)This is $7C3 = 7!/(4!3!) = (7 \times 6 \times 5)/(3 \times 2 \times 1) = 35$ because you are **selecting** 3 from 7.

(c) This is $7P3 = 7!/4! = 7 \times 6 \times 5 = 210$ because for the first day you have 7 choices, second day 6 choices, third day 5 choices.

(d) This is 3^7 because repetitions are allowed.

6. (Extra credit 10 points) You can partition the real numbers as a union of the subsets $[n, n+1)$. For example, $[-1, 0), [0, 1), [1, 2)$ are all members of this partition. What is the equivalence relation for which these subsets become the equivalence classes? Find the equation for the relation and prove that it is an equivalence.

   Solution: (This was done in class).

   The relation is $xRy \iff \lfloor x \rfloor = \lfloor y \rfloor$ where $\lfloor x \rfloor$ is the floor function given by $\lfloor x \rfloor =$ the greatest integer smaller than x.

   It is reflexive because floor of a number is equal to itself.

   If two numbers have the same floor value, order doesn't matter. So it is symmetric.

   If $\lfloor x \rfloor = \lfloor y \rfloor$ and $\lfloor y \rfloor = \lfloor z \rfloor$ then $\lfloor x \rfloor = \lfloor z \rfloor$. So it is transitive.