Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 50 minutes

Please read the questions carefully before answering

1. (12 points) Check if the map $\mathbb{Z} \to \mathbb{Z}$ given by $f(m) = 2m + 1$ is 1-1 and onto. Prove your answer.

   (8 points) Find its inverse function if it exists. What is the domain of the inverse function?

2. (a) (12 points) Show that the following is an equivalence relation on $P(S)$, the set of subsets of $S = \{1, 2, 3, 4, 5, ..., 9, 10\}$. Note that the elements of $P(S)$ are the subsets, and NOT the numbers 1,2,3,....

   "Two subsets A and B are related iff they have same number of elements"

   (b) (8 points) Find the equivalence class of the empty set and the set $\{1\}$.

   (c) (5 points) Show that the equivalence classes are distinct : no two of them will share a common element.

3. (20 points) State whether true or false. You must prove it if you think it is true. If it is false, provide a counterexample.

   (a) The sequence $n^2 - n + 1$ where $n \geq 1$ is increasing.

   (b) All equivalence classes under an equivalence relation will have the same number of elements.

   (d) If a relation is not symmetric then it is antisymmetric.

(e) A partial order cannot be a total order.

4. (10 points) Find the GCD of 111 and 30 using Euclidean algorithm.

5. For the following problems, you MUST explain the steps you took to arrive at final answer.

   (a) (6 points) How many outfits consisting of one pair of shorts, one shirt and a hat can you make from 5 shirts and 3 hats and 4 pairs of shorts ?

   (b) (7 points) How many choices do you have if you are packing 3 shirts for a trip from a set of 7 different shirts? (You are selecting 3 shirts).

   (c) (6 points) How many ways can you dress on 3 days with different shirts if you have a total of 7 shirts? For example, Mon - Shirt A, Tue – Shirt B, Wed – Shirt C and so on. Basically, you are arranging the 7 shirts in 3 places.

   (d) (6 points) If you have just 3 different shirts to wear for 7 days, but you can repeat the same shirt, how many possibilities are there for the whole week? For example, you can wear shirt A for all 7 days, or it can be something like AABBBCC.

6. (Extra credit 10 points) You can partition the real numbers as a union of the subsets $[n, n+1)$. For example, $[-1, 0), [0, 1), [1, 2)$ are all members of this partition. What is the equivalence relation for which these subsets become the equivalence classes? Find the equation for the relation and prove that it is an equivalence.