Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 50 minutes

Please read the questions carefully before answering

1. (20 points) State the converse (4 points) , contrapositive (6 points) and negative (6 points) of the following statement about sets. (4 points) Is the converse true?

   If a set of integers is finite then it has a maximum element.

2. (20 points) Write the following statement and its negative in symbols. Denote the set of natural numbers by $\mathbb{N}$. You should write ALL of it in symbols.

   If A is a subset of the natural numbers then it has a least element (least element means something smaller than all the other elements).

3. (20 points)

   a. Write the contrapositive and prove it:

      If $xy > 100$ then either x or y is greater than 10.

   b. Prove using definition of subsets:

      If X, Y are any two sets, then $X \subseteq Y \iff \overline{Y} \subseteq \overline{X}$.

   [If X is contained in Y then complement of Y is contained in complement of X].

4. (20 points) Prove by cases: $2m^2 + 3n^2 = 40$ has no solutions in positive integers.

5. (20 points) Show using regular induction that for any natural number n, the following is true:

$$\frac{1}{2n} \leq \frac{1 \times 3 \times 5 \times ..... \times 2n - 1}{2 \times 4 \times 6 \times .... \times 2n}.$$

6. (10 points extra credit) Show using strong induction that for any natural number $n \geq 4$, the Fibonacci numbers defined by $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3, F_5 = 5, F_6 = 8, ..., F_n = F_{n-1} + F_{n-2}$ satisfy

$$F_n < \left(\frac{7}{4}\right)^{n-2}.$$

Note that it *doesn't* work for n up to 3. For example, $(7/4)^{3-2} = 1.75$ but $F_3 = 2$.