Howard University Math Department

1. (15 points) Check whether the following relation defined on the set of positive integers is reflexive, symmetric, transitive, antisymmetric, and partial order:

   $xRy \iff x$ divides y. In other words, y/x is also an integer.

   Solution:

   Reflexive because every integer divides itself. $x/x = 1$ and so xRx is true.

   Not symmetric because x can divide y but not vice versa. Example: 2 divides 4 but 4 does not divide 2. So xRy doesn't mean yRx.

   It is antisymmetric because if x divides y and y divides x then they have to be equal. x/y and y/x are reciprocals of each other, And reciprocal of a number equals itself only if the number is 1 or -1. You can see this by letting $n = 1/n$ and solving for n. Since we are only dealing with positive integers, -1 is not an option. So xRy and yRx means $x = y$.

   Transitive: If x divides y and y divides z then x divides z. So xRy and yRz means xRz. You can show this as follows: $y = xk$ and $z = ym$ means $z = (xk)m = x(km)$ and km is also an integer.

   You can also say $y/x = k, z/y = m \implies z/x = (z/y)(y/x) = mk$ is also an integer.

   It is a partial order because it is reflexive, antisymmetric and transitive.

2. (15 points) How many 5-letter strings can be formed from the letters ABCDE if repetitons are allowed? How many can be formed from the letters ABCDE if repetitons are not allowed? How many starting with the letter A can be formed from the letters ABCDE if repetitons are not allowed?

   Solution:

   Each letter can be one of 5. So if repetitions are allowed, we can make $5 \times 5 \times 5 \times 5 \times 5 = 5^4 = 625$ strings.

   If repetitions are not allowed, we can make $5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$ strings.

   If A has to be first letter, remaining letters can be chosen in $4 \times 3 \times 2 \times 1 = 4! = 24$ ways.

3. (10 points extra credit) If the integers from 1 to 9 are partitioned as the union of the disjoint sets $\{1, 4, 7\}, \{2, 5, 8\}, \{3, 6, 9\}$ what are the equivalence classes? What is the equivalence relation?

   Solution:

The equivalence classes are the subsets themselves.

$[1] = [4] = [7] = \{1, 4, 7\}, [2] = [5] = [8] = \{2, 5, 8\}, [3] = [6] = [9] = \{3, 6, 9\}$

The relation is given by xRy iff 3 divides $x - y$.