Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (a) (8 points) Show that the following is 1-1 from the nonzero real numbers to all real numbers, i.e, $f : \mathbb{R} - \{0\} \to \mathbb{R}$.

   (b) (7 points) Find its inverse:

   $$f(x) = \frac{2-x}{x}.$$

   1-1: Show that $f(a) = f(b)$ implies $a = b$. Note that a, b are nonzero so we can 'cancel' them out on both sides.

   $$f(a) = \frac{2-a}{a} = \frac{2-b}{b} \implies 2b - ab = 2a - ab \implies a = b.$$

   To find inverse, set $f(x) = y$ and solve for x.

   $$\frac{2-x}{x} = y \implies \frac{2}{x} - \frac{x}{x} = y \implies \frac{2}{x} = y + 1 \implies x = 2/(y+1).$$

   Since a function is usually with input x, we switch variables and say $f^{-1}(x) = 2/(x+1)$.

   Check that $f(f^{-1}(x)) = x$.

2. (10 points) Is the function in problem 1 onto?

   To be onto, each real number y should have a pre-image x. From (1) we see that pre-image of y is $2/(y+1)$. If $y = -1$, this has no solution. So it is not onto.

3. (10 points) Find the GCD of 35 and 10 using Euclidean algorithm.

   $$35 = 3(10) + 5$$
   $$10 = 2(5) + 0$$

   So 5 is the GCD.