

1. (10 points) Write the following statement and its negative using nested quantifiers, given that domain of discourse is set of integers times itself ($\mathbb{Z} \times \mathbb{Z}$).

“Given any integer m , there is another integer n that is bigger than m .”

$$\forall m \exists n (n > m)$$

Negative:

$$\exists m \forall n (m \geq n)$$

“There is an m that is bigger than every other integer n ”

2. (10 points) Prove directly: If x is even and y is even then xy is even. (NOT enough to give examples!)

$$x = 2m, y = 2n \implies xy = 2m \times 2n = 2(2mn)$$

So xy is of the form 2 times something, hence it is even.

3. (10 points) Prove using contrapositive: If x^2 is irrational, then x is irrational.

Contrapositive: If x is rational, then x^2 is rational. This is true because if $x = p/q$ with p, q being integers, $q \neq 0$, then $x^2 = p^2/q^2$ which is also a rational number.