

1. (10 points) State the converse, contrapositive and negative of the following statement:
“If a natural number is even then 2 divides it.”

Converse: If 2 divides a natural number, then it is even.

Contrapositive: If 2 doesn't divide a natural number, then it is odd.

Negative: A natural number is even and 2 doesn't divide it.

2. (10 points) Check the validity of the following argument. You only need to check if the conclusion follows from the first statement based on the argument. Initial statement might be assumed to be true.

“If you go to Howard then you will succeed. You didn't succeed. So you didn't go to Howard.”

Solution: True (if original statement is). The contrapositive of the statement is “If you didn't succeed then you didn't go to Howard.” Since contrapositive is true if the given statement is, this is a valid argument.

3. (10 points) Write the following symbolic statement in words. If it is false, prove by counterexample. The domain of discourse is $\mathbb{R}$, the set of real numbers.

$$\forall x, x^2 \geq x.$$

Solution:

This says “For every real number, its square is bigger than or equal to itself.” This is false. Counterexamples are numbers strictly between 0 and 1, like for instance $1/2$. Its square is $1/4$ which is smaller than itself.

So we have proved the negative, which is $\exists x, x^2 < x$.