DISCRETE STRUCTURES 10/9/26 CLASSWORK

SEQUENCES AND STRINGS

SOME DEFINITIONS

Let s_n be the terms of a sequence. n is called the INDEX of the sequence

Increasing : $s_n < s_{n+1}$

Decreasing : $s_n > s_{n+1}$

Nondecreasing : $s_n \leq s_{n+1}$

Nonincreasing : $s_n \geq s_{n+1}$

Sum of a sequence: $\sum_{\{i=1\}}^{k} s_n$ means $s_1 + s_2 + \ldots . + s_k$

Product of a sequence: $\prod_{\{i=1\}}^{k} s_n$ means $s_1 \times s_2 \times \ldots . \times s_k$

Subsequence is just a sequence within a sequence, where the indices are a subset of the indices of original sequence.

For example if we take the odd index terms of the sequence 1, 2, 4, 8,…2^{n-1}, … we get the subsequence 1, 4, 16,….

What is the k-th term of this subsequence?

Is the original sequence increasing?

Write the sum of the first m terms starting with the index i = n-1 instead of n.

ANSWERS:

k-th term of subsequence is obtained by letting n = 2k-1 because 2k-1 gives the odd numbers starting with 1.
So n-1 = 2k-2.

We get $a_k = 2^{2k-2}$. Check: k = 1 gives 1, k=2 gives $2^2 = 4$, k=3 gives $2^4 = 16$, and so on.

Yes, original sequence is increasing.
Proof: $s_n < s_{n+1}\ because\ 2^{n-1} < 2^{(n+1)-1} = 2^n$

If i = n-1 then for n=1 we get i = 0 when n=1, and i = m-1 when n = m.

The sum looks like $\sum_{\{i=0\}}^{\{m-1\}} 2^i$

Example of a non-decreasing sequence: 3.2, exercise 53-56: $n^2 - 3n + 3$

Show that $s_1 = s_2 \; but \; s_n < s_{n+1} \; for \; n \geq 2$
So it is non-decreasing. It is not increasing if you consider the whole sequence, but if you look at only the terms starting with s_2 then it is increasing.

A STRING is just a finite sequence, usually written together.
For example, the sequence a, b, c, d, e gives rise to the string abcde.
Bit strings are sequences of 1's and 0's, used in computers.
Two strings can be *concatenated* by just writing them next to each other.