10/16/2026 DISCRETE STRUCTURES

QUIZ 6 ON MONDAY

TODAY: EQUIVALENCE CLASSES

R is an equivalence relation on a set S if (1) reflexive (2) Symmetric (3) Transitive

Example: R is a relation on $\mathbb{Z}$ given by

xRy iff $3 \mid (x-y)$

— same as saying $x \equiv y \bmod 3$

x & y have same remainder when divided by 3

example: 5 & 2 are related: $3 \mid (5-2)$

Reflexive: xRx $\forall x \in \mathbb{Z}$? $3 \mid x-x$? $3 \mid 0$ ✓

Symmetric: If xRy then yRx?

$3 \mid (x-y)$ means $3 \mid (y-x)$

If $x-y = 3k$ then $y-x = -3k = 3(-k)$

Transitive: If xRy & yRz then xRz

$x-y = 3k_1$ & $y-z = 3k_2 \Rightarrow x-z = 3(k_1+k_2)$

so xRz also

Equivalence classes

Equivalence class of an element $x \in S$

If R is equivalence relation on S, then

$[x] = \{y \in S \mid xRy\}$ = Set of elements related to x

NOTE: x is always in $[x]$ because xRx by reflexivity of R

Look at [1] under R given by xRy iff $3 \mid (x-y)$

$[1] = \{y \in S \mid 1Ry\} = \{y \in S \mid 3 \mid (1-y)\}$

$= \{y \in S \mid 1-y = 3k$ for some $k \in \mathbb{Z}\}$

$= \{y \in S \mid y = 3k+1$ for $k \in \mathbb{Z}\}$

$= \{\ldots, -8, -5, -2, 1, 4, 7, \ldots\}$

NOTE: $[4] = \{y \in S \mid 4Ry\} = [1]$ because $4 \equiv 1 \bmod 3$

Equivalence class of any element in [1] is the same set [1]

All the elements in an equivalence class have the same equivalence class

---

Example 2 Let $S = \{f : \mathbb{R} \to \mathbb{R}\}$

R is given by $fRg \iff f(0) = g(0)$

Reflexive? $f \in S \Rightarrow fRf$? Yes. $f(0) = f(0)$

Symmetric? $fRg \Rightarrow gRf$? Yes. $f(0) = g(0) \Rightarrow g(0) = f(0)$

Transitive? fRg & $gRh \Rightarrow fRh$?

$f(0) = g(0)$ & $g(0) = h(0)$

$\Rightarrow f(0) = h(0)$.

What is the equivalence $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x$?

$[f] = \{g \in S \mid fRg\} = \{g \in S \mid f(0) = g(0)\}$

$= \{g \in S \mid 0 = g(0)\}$

$g(0) = 0$ means g passes thru $(0, 0)$

