Howard University Math Department

Homework 2 Solutions(16 points)

1. Prove or disprove by counterexample: Given that the first n prime numbers are $p_1, p_2, p_3, ..., p_n$ the number $P_n = (p_1 \times p_2 \times ... \times p_n) + 1$ is also a prime.

   (For example, $2 \times 3 + 1 = 7, 2 \times 3 \times 5 + 1 = 31$ are both primes. Is this always true?)

   Solution:

   This is true for 2+1= 3,(2.3)+1= 7,(2.3.5)+1=31,(2.3.5.7)+1=211,(2.3.5.7.11)+1=2311 but not for (2.3.5.7.11.13)+1= 30031 which happens to be 59 times 509.

2. Write the negative, converse and contrapositive of the following statement:

   Statement A: If the product of three positive real numbers is ≥ 27, at least one of them is ≥ 3.

   Solution: Let first part statement be I and second part be II. Then A is $I \implies II$.

   Negative: $I \implies NOT\ II$.

   If the product of three positive real numbers is ≥ 27, then none of them is ≥ 3. (or all of them are < 3).

   Note that negative of "at least one" is "none." You can see that "at least one is blue" is not opposite of "atleast one is not blue." Both can be true at same time.

   Converse: $II \implies I$.

   If at least one of three positive real numbers is ≥ 3 then their product is ≥ 27.

   Contrapositive: $NOT\ II \implies NOT\ I$.

   If none of three positive real numbers is ≥ 3 then their product is < 27.

3. Prove statement A using proof by contrapositive.

   Solution: If all are less than 3, then we can multiply together $a < 3, b < 3, c < 3$ to get $abc < 27$ and inequality would be unchanged because all are positive.

4. Prove that converse is false.

   Solution: Proof by counterexample. Let $a = 3, b = 1, c = 1$. Then at least one is ≥ 3 but their product is smaller than 3, and not ≥ 3.