

# Proof and Problem Solving

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8/220/2018

Supplementary Class Notes

# Some definitions:

- Axiom: A statement that is accepted as true or assumed to be true and often used as the starting point in a mathematical system. These cannot be proved to be true starting from other assumptions.
- Equivalent statements: Two statements A and B are *equivalent* if the truth of A implies that B is true and vice versa

# Euclid's Fifth Axiom

*Through a given point there can be only one line parallel to another line that is not passing through that point.*

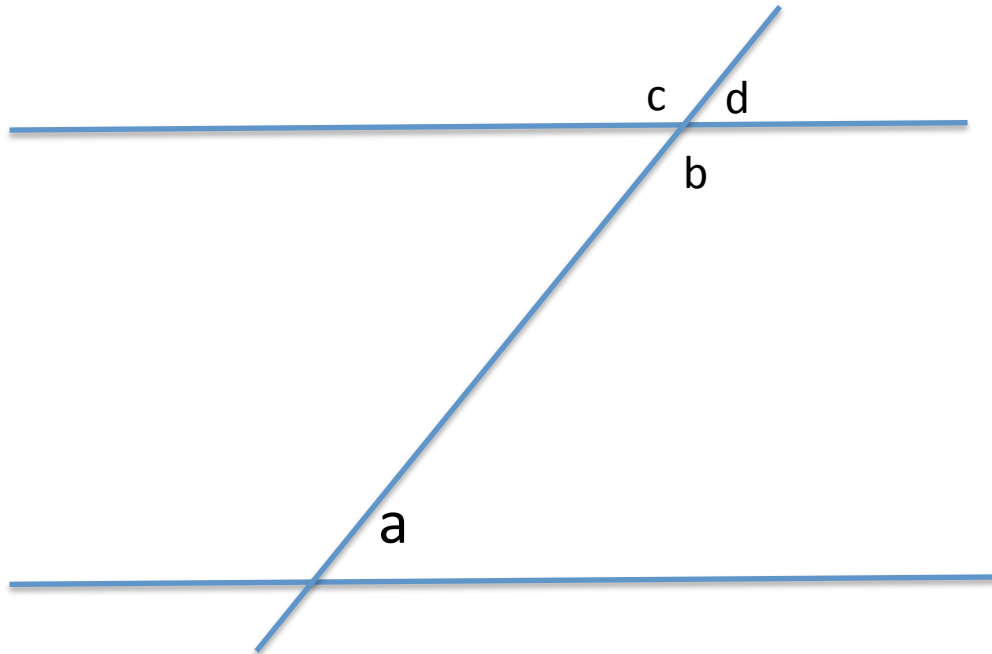
This is equivalent to each of the following :

1. The sum of the interior angles made by a line intersecting two parallel lines equals two right angles (=  $180^\circ$ ).
2. In any triangle the sum of all angles equals two right angles.

# Question 3

- Prove that sum of angles of a triangle is 180 (Euclid's fifth postulate) using statements A,B and C:
- (Statements A,B,C below can be proved using Euclid's first four axioms – see handout on update page)
- (A) Sum of the interior angles made by a line intersecting two parallel lines equals two right angles ( $=180^\circ$ )
- (B) Alternate angles made by a line intersecting two parallel lines are equal
- (C) Opposite angles made by two intersecting lines are equal
- See next page for picture explaining A,B,C.

# Explanation of A,B,C



Here we have two lines that are parallel and a line that intersects both. a and b are examples of interior angles ; b and c are examples of opposite angles; a and d are examples of alternate angles. Statement A means sum of angles  $a+b$  is 180 ; B means angles a and d are equal; C means b and c are equal.