TEST 2 REVIEW

PROOFS AND PROBLEM SOLVING 1, FALL 2025

1. Prove directly: A is a subset of B then AUB equals B

When are two sets equal? When each is a subset of the other.

To show: $A \subseteq B \Rightarrow A \cup B = B$

1) $A \subseteq B \Rightarrow A \cup B \subseteq B$

$x \in A \Rightarrow x \in B$ then $x \in A \cup B \Rightarrow x \in B$?

$\Downarrow$

$x \in A$ or $x \in B$ $\nearrow$ yes

2) $A \subseteq B \Rightarrow B \subseteq A \cup B$

$x \in B \Rightarrow x \in A \cup B$ True by definition

So $A \cup B = B$

2) If $Q_n = n!+1$, then $\exists\, p > n$, p prime, such that $p \mid Q_n$

If Q_n were prime itself, then done

If not, $\exists\, p \mid Q_n$, p prime

But $p \leq n \Rightarrow p \mid n! \Rightarrow p \nmid n!+1$

(because $p > 1$)

But we must have a $p \mid Q_n$. So $\exists\, p > n, p \mid Q_n$

3) Equivalence classes under relation $ARB \Leftrightarrow \exists\, f : A \to B$ 1-1, onto on the $P(\mathbb{Z})$ (set of subsets of $\mathbb{Z}$)

Example: What is the equivalence class of $\{\}$?

$(\{\}) = ?$

$AR\{\} \Leftrightarrow \exists f: A \to \{\}$, 1-1 onto

A function must have a unique image for each element in domain

So $(\{\}) = \{\}$ (vacuously)

$(\{1\}) = ?$ $f: A \to \{1\}$, 1-1, onto

Then A must be of form $\{n\}$ with $n \in \mathbb{Z}$

$(\{1\}) = \{\{n\} \mid n \in \mathbb{Z}\}$

$(\mathbb{Z}) = \{$All infinite subsets$\}$