Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (15 points) State the converse, contrapositive and negative of the following statement about functions taking real numbers to real numbers that are differentiable in a closed interval $[a, b]$: (Need not check if statement is true)

   If the function $f(x)$ has a local maximum or minimum in $[a, b]$ then the derivative is zero for some $x \in [a, b]$.

2. (16 points) Use truth table to check that $p \implies q$ and $(\neg p) \vee q$ are equivalent.

3. (16 points) State the negative of the following statements:

   a) Either the climate is not changing or most scientists are lying.

   b) $\forall x \in [0, \infty), x^2 \leq 2^x$.

4. (16 points) Prove using the contrapositive: If x is a rational number and y is irrational, then

$x + y$ is irrational.

5. (16 points) Prove or give counterexample: Domain is set of positive real numbers.

$$\forall x \in (0, \infty),\ 2^x - 1 \leq x^2. \quad (Note:\ x > 0).$$

6. Given $A = \{1, 2, 3, 4\},\ B = \{3, 4, 5\},\ C = \{1, 5, 6\}$ and the universal set is $U = \{1, 2, 3, 4, 5, 6\}$ check if following are true *for A, B , C.*

   NOTE: To disprove something, enough to give ONE counterexample. So if a statement is not true for some sets, it is false. But to establish it to be true, need to prove for ALL sets. So you can only disprove something using the specific sets A, B, and C.

   a) (8 points) $A \cup (B \cap C) = (A \cup B) \cap C$.

   (b) (6 points) $\overline{A \cup C} = \overline{A} \cap \overline{C}$

   (c) (8 points) $A - (B \cap C) = (A - B) \cup C$