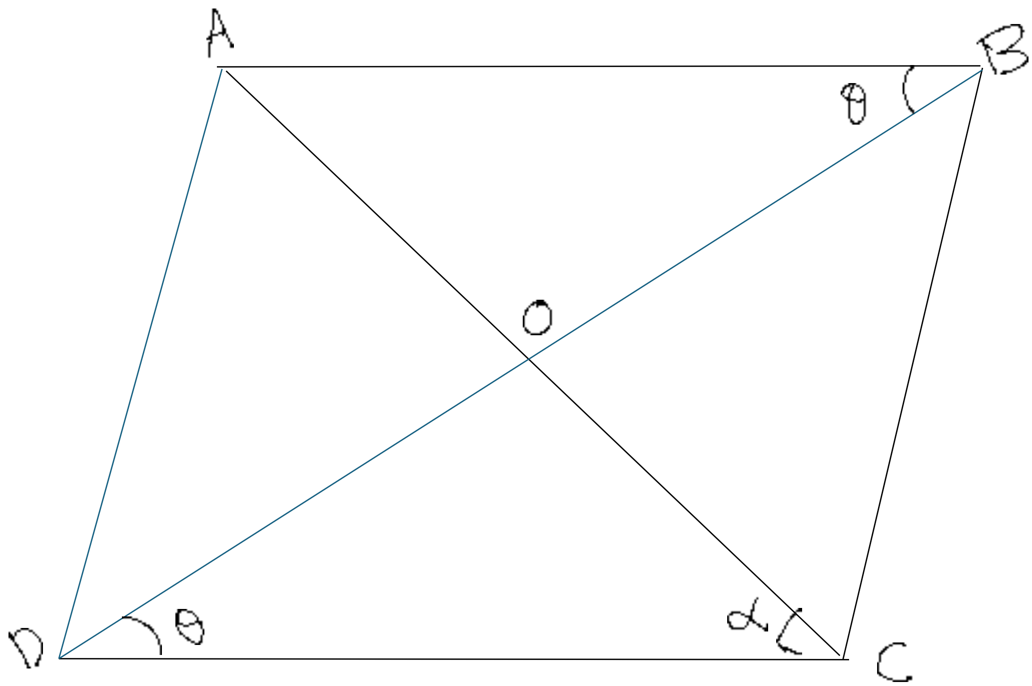
SOME FACTS ABOUT RHOMBUS (AB=BC=CB=DA)



PROVE:

1) All angles at O equal 90 degrees

2) If $\theta = \alpha$ then it is a square

You can assume:

1) Opposite sides are parallel

2) Angles made by a line on parallel sides such as BDC (θ) and ABD are equal.

3) Angles in a triangle (or on a line) add up to 180 and angles around a point add up to 360.

4) Angles opposite equal sides in an isosceles triangle are equal.

Proof idea:

The triangle ABD is isosceles because AB = BC, and so the angles opposite them are equal. This means the angle ADB (the other angle at the corner D) is also θ.

The angle CAB (at the corner A) is equal to the angle ACD (which is α) because they are the interior opposite angles both made by the diagonal AC on the parallel lines AB and DC.

Using the isosceles nature of ADC, ABC and BDC and the interior opposite angles made by the diagonal DB we can show that the angles at the corner B are both θ, and the angles at the corner C as well as at the corner A are both α.

Now take the triangle DAB. Its angles are θ, θ, and 2α. Their sum equals 180. So we have $2\theta + 2\alpha = 180$. From this we get $\theta + \alpha = 90$. From this you should be able to prove that the angles at O are all 90, and if $\theta = \alpha$ then all the corner angles are 90 which makes it a square.

ALTERNATIVE WAYS TO PROVE:

You could compare the four triangles which have O as a vertex, namely AOB, BOC, COD, DOA. As shown above, they all have angles 90, θ and α and one side (the side of the rhombus that is part of the triangle) and since all angles are same and one side is equal they are congruent. Now, comparing sides opposite the same angles, we see that the diagonals cut (bisect) each other into equal intervals. In other words, AO = OC and BO = OD. (This last fact is not really necessary).

Now, the angle AOB equals the angle DOC because the corresponding triangles are congruent and these two angles are opposite two sides that are equal in length, namely AB and DC. (Actually one can prove that at the intersection of two lines the vertically opposite angles are equal). Similarly angle BOC = angle DOA.

So if we can show that one from each pair, say the angles AOB and BOC are equal, then all four are equal, and since they add to 360, each equals 90.

To show AOB and BOC angles are equal, use the fact that the corresponding triangles are congruent and note that the two angles are opposite two sides that are equal in length, namely AB and BC.