Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (18 points) Check which of the following relations between the real numbers $\mathbb{R}$ and the integers $\mathbb{Z}$ are well-defined functions. If it is a well-defined function, say if it is 1-1 and if it is onto. Explain your answers.

   (a) The floor function : $f(x)$ is the greatest integer $n \leq x$.

   Example: $f(1.1) = 1, f(-3.7) = -4, f(0) = 0$, etc.,

   (b) $f(x) = \dfrac{x}{|x|}$.

   (c)(Codomain is also real numbers for this) $f(x) = y$ where y is found by solving the equation $x = y^2$

2. (16 points) Find the range of the following functions, and check if they are onto. Explain.

   (a) $f : \mathbb{R} \to \mathbb{R},\ f(x) = x^3$.

   (b) $f : \mathbb{R} \to \mathbb{R},\ f(x) = e^x$.

3. (20 points) Prove the following statements if true. Give counterexample if false.

   a) The composition of two injective (i.e, 1-1) functions is always injective.

   b) A 1-1 function from a set to itself is always onto.

4. (15 points) Prove the following: If R is an equivalence relation on a set X and $Y \subseteq X$ is any subset of X then R is also an equivalence relation on Y. Prove also that the equivalence classes of Y are of the form $Y \cap X_i$ where X_i are the equivalence classes of X.

5. For the relation R defined on the set of integers $\mathbb{Z}$ by $mRn \iff 7\ divides\ m-n$ prove the following:

   a) (10 points) Prove that it is an equivalence relation.

   (b) (6 points) Find the equivalence class of 1. Describe the set using set notation.

   (c) (8 points) How many equivalence classes are there? Describe all of them using set notation.

   (d) (7 points) Show that the equivalence classes are distinct and they cover all integers.

6. (Challenge: Extra credit 10 points) Check if the following relation R defined on the set of rational numbers S whose denominators are not divisible by 3 is reflexive, symmetric, and transitive. Here the fractions are written after canceling off common factors. For example,in a/b, a and b have no common factors, and 3 does not divide b and similarly for c/d and all other fractions:

$$\frac{a}{b} R \frac{c}{d} \iff 3\ divides\ ad-bc.$$