2/11/2026 Spring 2026, Proofs and Problem Solving Test 1 Solutions Sitaraman
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1. (15 points) State the converse, contrapositive and negative of the following statement about functions taking real numbers to real numbers in a closed interval $[a, b]$:

   If a function $f(x)$ is continuous in $[a, b]$ then the integral $\int_a^b f(x)\ dx$ exists.

   Converse: If the integral of $f(x)$ exists in $[a, b]$ then the function f is continuous.

   Contrapositive: If $\int_a^b f(x)\ dx$ does not exist then the function f is not continuous in $[a, b]$.

   Negative: Some function $f(x)$ is continuous in $[a, b]$ and $\int_a^b f(x)\ dx$ doesn't exist.

   NOTE: **Negative of** $A \implies B$ **is** $A \wedge \neg B$.

2. (16 points) Use truth table to check that $p \implies q$ and $p \wedge \neg q$ are negatives of each other.

   Solution:

p	q	$p \implies q$	$p \wedge \neg q$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

   One is T when the other is F and vice versa, so they are negatives of each other.

3. (18 points) State the negative of the following statements:

   a) Either it is freezing or it is too hot.

   b) $\forall x \in (-\infty, \infty), (x+1)^2 > 2x$.

   c) $\exists x \in \mathbb{R}, (x+1)^2 \leq 2x$ and $x \geq 0$.

   Solution:

   a) It is neither freezing nor too hot.

   b) $\exists x \in (-\infty, \infty), (x+1)^2 \leq 2^x$.

   c) $\forall x \in \mathbb{R}, (x+1)^2 > 2x$ or $x < 0$.

4. (15 points) Prove using the contrapositive: If m is an integer and $m+n$ is not an integer, then n is also not an integer.

   Solution:

   Proof: Assume n is an integer. We need to show that either m is not an integer or $m+n$ is an integer. If m is not, we are done. Otherwise, m, n are both integers and $m+n$ is an integer. So we are done again.

5. (12 points) Prove that the sum of the angles at the vertices of a pentagon is 540 degrees. The pentagon need not be regular, i.e, the lengths of the sides could be different. Assume the pentagon is "nice" (convex, for example).

   One way to prove it: Use the fact that the sum of angles in a triangle is 180, and that when you go once around the center of the pentagon you cover 360 degrees.

   (Extra credit 5 points) Can you come up with a formula for the sum of angles in a polygon with n sides? In other words, the same formula must work for triangle, quadrilateral, pentagon, hexagon, etc., if you plug in the corresponding value of n.

   Solution:

   One way: Join each vertex to the center by a line segment. You get 5 triangles. Total of all the angles in the five triangles is 180 times 5 or 900. Of these, 360 degrees come from the angles at the vertex. Subtracting, we get 540 degrees.

   Another way: Divide pentagon into 3 triangles by joining alternate vertices. Then the total angle at the vertices will be the same as the total of the angles in the 3 triangles which is 3 times 180 or 540 degrees.

   In general, the total of all the angles in an n-gon is $(180 \times n) - 360 = 180(n-2)$ using the first method.

   Using second method, you get $n-2$ triangles for an n-gon, so total angle is $(n-2) \times 180$.

6. Given $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, $C = \{1, 5, 6\}$ and the universal set is $U = \{1, 2, 3, 4, 5, 6\}$ check if following are true *for* A, B, C.

   NOTE: If something is true for the specific A, B, C it doesn't mean it is true for all sets A, B, C but you can disprove something with just ONE counterexample. So if a statement is not true for these sets, it is a false statement.

   a) (8 points) $A \cup (B \cup C) = (A \cup B) \cup C$.

   (b) (8 points) $\overline{A \cap C} = \overline{A} \cup \overline{C}$

   (c) (8 points) $A - (B - C) = (A - B) - C$

   Solution:

   a) $A \cup (B \cup C) = \{1, 2, 3, 4, 5, 6\} = (A \cup B) \cup C = \{1, 2, 3, 4, 5, 6\}$.

(b) is true by DeMorgan's law.

(c) $A - (B - C) = A - \{3, 4\} = \{1, 2\}$ and it is $\neq (A - B) - C = \{1, 2\} - C = \{2\}$.

7. (Extra credit 5 points) Is the converse of statement in problem 1 true?

   Solution:

   False. For instance you can integrate a step function like $f(x)$ given by $f(x) = 1$ for $x \in [0, 1]$ and $f(x) = 2$ for $x \in (1, 2]$ and its integral will be 3. But the function is not continuous because it has a jump at 1.