

Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (15 points) State the converse, contrapositive and negative of the following statement about functions taking real numbers to real numbers in a closed interval $[a, b]$:

If a function $f(x)$ is continuous in $[a, b]$ then the integral $\int_a^b f(x) dx$ exists.

2. (16 points) Use truth table to check that $p \implies q$ and $p \wedge \neg q$ are negatives of each other.

3. (18 points) State the negative of the following statements:

a) Either it is freezing or it is too hot.

b) $\forall x \in (-\infty, \infty), (x + 1)^2 > 2x$.

c) $\exists x \in \mathbb{R}, (x + 1)^2 \leq 2x$ and $x \geq 0$.

4. (15 points) Prove using the contrapositive: If m is an integer and $m + n$ is not an integer, then n is also not an integer.

5. (12 points) Prove that the sum of the angles at the vertices of a pentagon is 540 degrees. The pentagon need not be regular, i.e, the lengths of the sides could be different.

One way to prove it: Use the fact that the sum of angles in a triangle is 180, and that when you go once around the center of the pentagon you cover 360 degrees.

(Extra credit 5 points) Can you come up with a formula for the sum of angles in a polygon with n sides? In other words, the same formula must work for triangle, quadrilateral, pentagon, hexagon, etc., if you plug in the corresponding value of n .

6. Given $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, $C = \{1, 5, 6\}$ and the universal set is $U = \{1, 2, 3, 4, 5, 6\}$ check if following are true *for* A , B , C .

NOTE: If something is true for the specific A, B, C it doesn't mean it is true for all sets A, B, C but you can disprove something with just ONE counterexample. So if a statement is not true for these sets, it is a false statement.

a) (8 points) $A \cup (B \cup C) = (A \cup B) \cup C$.

(b) (8 points) $\overline{A \cap C} = \overline{A} \cup \overline{C}$

(c) (8 points) $A - (B - C) = (A - B) - C$

7. (Extra credit 5 points) Is the converse of statement in problem 1 true?