Howard University Math Department

Any partition of a set gives an equivalence relation

Suppose a set X is partitioned by the collection of disjoint subsets X_i. This means every element of X belongs to one and only one of the X_i.

$$X = \bigcup_i X_i,\ X_i \cap X_j = \{\}\ if\ i \neq j.$$

Define a relation R on X by

$$xRy \iff x, y \in X_i \text{ for some } i.$$

In other words, x, y are rleated if both belong to one of the X_i.

Prove that R gives an equivalence relation on X.

Solution:

Reflexive: Since X is a union of the X_i, $x \in X_i$ for some i for all $x \in X$, so xRx for all $x \in X$.

Symmetric: If xRy then $x, y \in X_i$ for some i. Then $y, x \in X_i$ as well. So yRx also.

Transitive: If xRy and yRz then $x, y \in X_i$ and $y, z \in X_j$. But then $y \in X_i \cap X_j$ and that means $X_i = X_j$. This means x, z are in the same subset and so xRz as well.