Howard University Math Department

Finding orbit of a relation

We have an invertible (hence 1-1) function from the set $(-1,1)$ to $(-\pi/2,\pi/2)$ given by $g(x) = (\pi/2)x$ (prove it is 1-1 and onto using definition).

Then you have another 1-1, onto map given by $f(x) = \tan x$ from $(-1,1)$ to $\mathbb{R}$

You can see it is 1-1 using horizontal line test on graph of $\tan x$. You can see it is onto because each y value comes from some x value as you can see in graph. The horizontal line through each point on y-axis cuts the graph once, and only once.

Since $\tan((\pi/2)x) = f \circ g(x)$, and compositions of 1-1 onto functions are also 1-1, onto (as shown in notes and in test) we have a bijection from $(-1,1)$ to $\mathbb{R}$.

NOTE: You can prove everything directly by looking at graph of $\tan((\pi/2)x)$ instead of $\tan x$. Just say that each horizontal line cuts graph once and only once on that graph. This way you don't need $g(x)$.

Graph of $\tan x$