Howard University Math Department

The first 3 problems involve the following data:

These are the amounts of wind-power produced by the top ten states for wind power in the US, rounded to the nearest Gigawatt, in January 2026 (One GigaWatt can power more than 200,000 homes).

$Texas$	11
$Iowa$	4
$Oklahoma$	4
$Illinois$	3
$Kansas$	3
$Wyoming$	2
$Colorado$	2
$N.Dakota$	2
$Minnesota$	1
$Indiana$	1

1. (20 points) Find the following for the amounts of wind-power:

   (a) average (b) the standard deviation.

   You must show the formulas and all the steps involved.

   Solution:

   (a) There are 10 data points. So to find the average μ we need to add them and divide by 10.

$$\mu = \frac{33}{10} = 3.3.$$

   (b)The standard deviation σ is given by

$$\sqrt{\frac{(11-3.3)^2+(4-3.3)^2+(4-3.3)^2+(3-3.3)^2+(3-3.3)^2+(2-3.3)^2+(2-3.3)^2+(2-3.3)^2+(1-3.3)^2+(1-3.3)^2}{10}}$$

$$= \sqrt{\frac{7.7^2 + 0.7^2 + 0.7^2 + (-0.3)^2 + (-0.3)^2 + (-1.3)^2 + (-1.3)^2 + (-1.3)^2 + (-2.3)^2 + (-2.3)^2}{10}}$$

$$= \sqrt{\frac{59.29 + 0.49 + 0.49 + 0.09 + 0.09 + 1.69 + 1.69 + 1.69 + 5.29 + 5.29}{10}}$$

$$= \sqrt{7.61} \simeq 2.7586.$$

2. (15 points) Find the median, the mode, and the value in the 65th percentile.

   The median is the average of the 5th and 6th data points, namely $(2 + 3)/2 = 2.5$.

The mode is 2 since it appears 3 times, more than any other data point.

The 65th percentile means the rank of the value is 6.5 out of 10 values. 3 appears in seventh and sixth position, so the position of 3 is 6.5. Thus 3 has rank 6.5, so it is in 65th percentile.

3. (10 points extra credit)

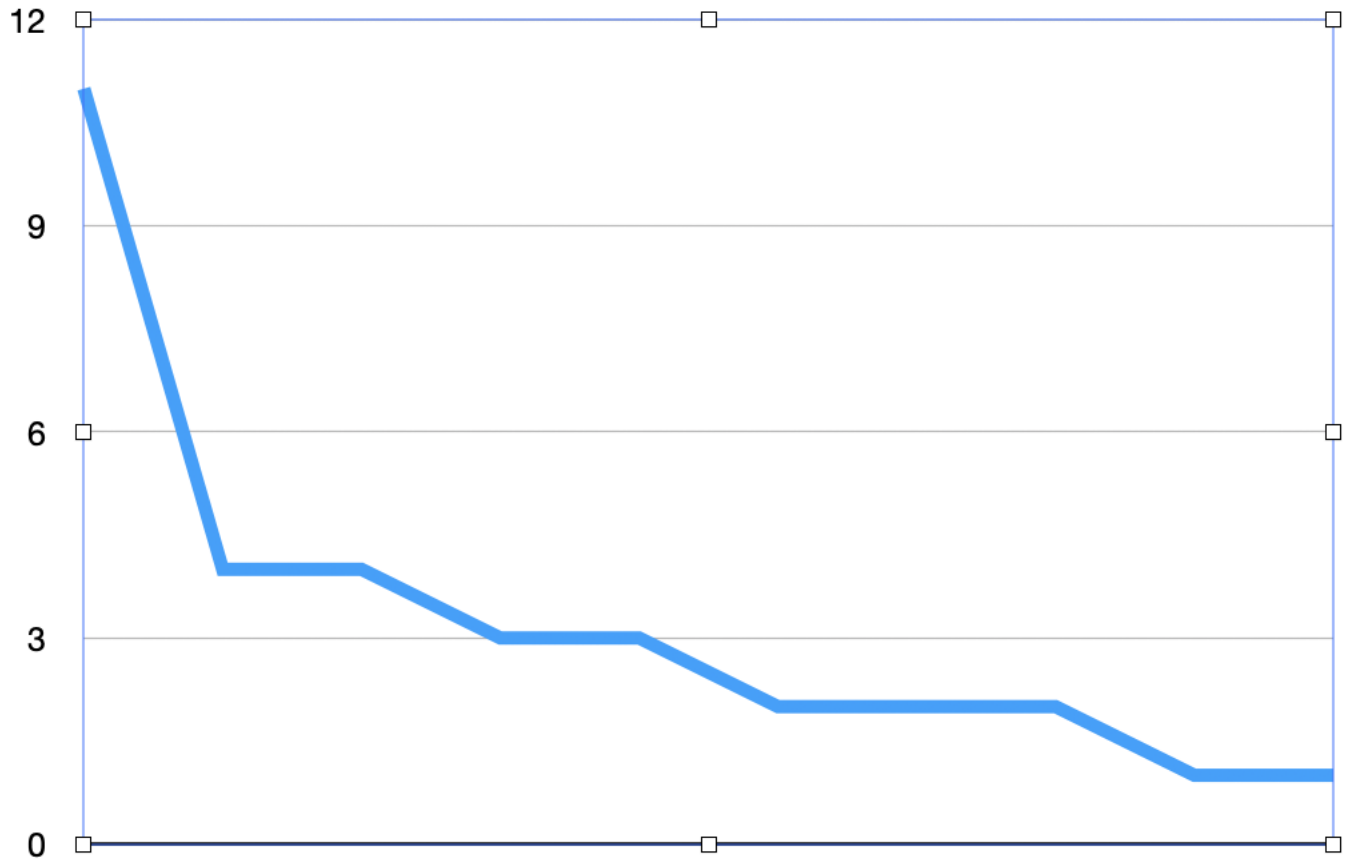
   a) In the problem above, why is the average more than the median?

   b) What does the standard deviation tell about the distribution of the data? Are they spread out or mostly near the average ? Could it be normal distribution?

   Solution:

   The average is higher because of the one big value, namely 11.

   9 out of 10 or 90 percent of the values are within one standard deviation of the average, that is, between $3.3 + 2.7586 = 6.0586$ and $3.3 - 2.7586 = 0.5414$. So it is **more bunched up than the normal distribution**. So it cannot be normal. Not very spread out, overall. You can see in the picture below, most values are near the average.



4. (15 points) For the following sequence, find the common ratio, the formula for the n-th term and calculate the sum of the first 100 values using the formula for the sum of a geometric

sequence. (Leave your answers as powers).

$$1,\ 3,\ 9,\ 27, ..., \ldots$$

Solution:

This is a geometric sequence because each time we are multiplying by the same number 3.

So the common ratio is $r = 3$ and the first term $a = 1$.

$$a_n = 1(3^{n-1}) = 3^{n-1}$$

$$a_1 + a_2 + a_3 + ... + a_{100} = a \times \frac{r^n - 1}{r - 1} = 1 \times \frac{3^{100} - 1}{3 - 1} = \frac{3^{100} - 1}{2}.$$

5. (10 points) If two coins are tossed, find $P(HH)$.

   Then using $P(HH)$ find the probability P(at least one tail).

   Solution: There are 4 outcomes : HH, HT, TH, TT.

   $P(HH) = 1/4$. Now probability of an event not happening is 1 minus probability of it happening. If it is two heads then it cannot have at least one tail.

   Therefore, P(at least one tail) $= 1 - (P(HH)) = 1 - (1/4) = 3/4$.

6. (30 points) (a) Find the number of ways to select color of pants, color of shirt and color of hat from 7 colors, if they should be of different colors.

   (b) Find the number of ways to select color of pants, color of shirt and color of hat from 7 colors, if they can be of same color.

   (c) Find number of ways to pick 3 colors from 7 without regard to what goes where.

   Soln:

   (a) No repetitions, order matters because different colors go to different parts of the outfit. So number of ways is $7P3 = 7 \cdot 6 \cdot 5 = 210$.

   (b)Repetitions are allowed. $7 \times 7 \times 7 = 343$.

   (b) No repetitions, but order doesn't matter because we just are picking 3 colors, without deciding which dress is what color. So number of ways is $7C3 = 7P3/(3!) = 210/6 = 35$.

7. (10 points) Suppose you are trying to guess what color combination your friend is wearing, and you know your friend always matches the colors of the pants and shirt but the hat could be any one of the 7 colors regardless of the color of the pants and shirt. What is the probability that you guessed the right combination?

   Solution:

   There is only one right combination. There are 7 colors for pants and shirts, and 7 possible colors of hat. So there are 49 possible combinations and the probability you guessed right is 1/49.

   You can also say the two events are independent. You can multiply the probabilities of getting the color of the pants and shirt right with the probability of getting the color of the hat right. Both are 1/7 so you get the combined probability equals $(1/7) \times (1/7) = 1/49$.