3/18/2026 Spring 2026, Patterns in Environmental Mathematics
Test 2 Solutions Sitaraman

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1. (25 points) For the parabola given by $y = 2x^2 - 4x$ find the following:
   (a) *Equation of* the axis of symmetry.
   (b) The coordinates of the vertex.
   (c) The x-intercepts.
   (d) The y-intercepts.
   (e) The graph of the paarabola with all of the above labelled clearly.

Solution:

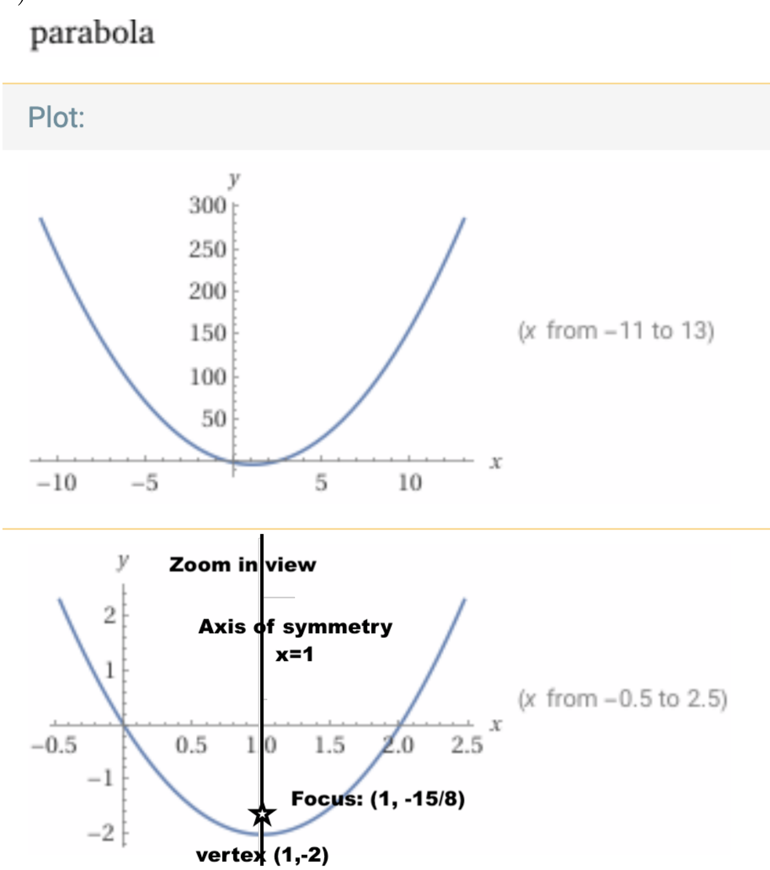
a) Equation of axis is $x = -b/(2a) = -(-4)/(2 \times 2) = 1$ so it is $x = 1$.

b) Vertex is given by $x = 1, y = 2(1^2) - 4(1) = -2$ so it is $(1, -2)$.

c) x-intercept is given by $y = 0$ and solving. $y = 2x^2 - 4x = 0 \implies x(x-2) = 0$ after canceling out 2 and factoring out x. Now $x(x-2) = 0 \implies x = 0, x = 2$ are the solutions.

d) The y-intercept is given by $x = 0$ which gives $y = 2(0^2) - 4(0) = 0$.

e)



2. (10 points extra credit)

   a) In the parabola of problem 1, where is the focus located? Mark it in your graph and write down the coordinates.

   b) If the Profit (or loss when the value is negative) from a business is $2x^2 - 4x$ million dollars when x units are sold, what are all the values of x that give a profit? Note that this is the same expression as in problem 1.

   Solution:

   The focus is located at a distance of $1/(4a) = 1/8$ from the vertex, inside the parabola, as shown in picture. Its coordinates are $(1, -2 + (1/8)) = (1, -15/8)$.

3. (15 points) For the following sequence, find the common ratio, the formula for the n-th term and the value of the 100th term using the formula for the geometric sequence : (Leave your answers as powers).

$$\frac{1}{3},\ 1,\ 3,\ 9,\ 27, ..., \ldots$$

   Solution:

   This is a geometric sequence because each time we are multiplying by the same number 3.

   So the common ratio is $r = 3$ and the first term $a = 1/3$.

$$a_n = \frac{1}{3}(3^{n-1}) = \frac{3^{n-1}}{3^1} = 3^{n-2}\ ;\ \ a_{100} = \frac{1}{3}(3^{100-1}) = \frac{3^{99}}{3^1} = 3^{98}.$$

4. (20 points) Given that 1 million dollars are invested in a fund that returns 8 percent annually, find the following. You must use compound interest formula. Leave your answers as millions of dollars, for example 1 million and not 1,000,000.

   (a) Equation for the amount after t years.

   (b) The amount after 10 years.

   (c) The time it would takes to double. You must use logarithms.

   (d) The time it would take to quadruple. [Hint: Easy if you use the answer in (c)].

   Solution

   (a) $A(t) = 1(1.08^t) = 1.08^t$

   (b) $1.08^{10} = 2.158925$ milions.

   (c) $2 = 1.08^t \implies t = \ln 2/\ln 1.08 = 9.0064$ years.

   (d) The 2 million dollars becomes 4 million in another 9.0064 years, so totally 18.0128 years to quadruple.

   The key point to remember is that doubling time is the same, regardless of what amount you start with.

5. (15 points) The initial amount is 1 million, annual rate of interest is 8 percent, as above. You must use compound interest formulae.

   a) Find the amount after t years if interest is compounded twice a year.

   b) Find the amount after 9 years if interest is compounded twice a year.

   c) Find the amount after 9 years if interest is compounded continuously.

   Solution:

   a) We have $P = 1, r = 0.08, n = 2$.

$$A(t) = 1\left(1 + \frac{r}{n}\right)^{nt} = (1.04)^{2t}.$$

   b) For this put $t = 9$ in above formula: $A(9) = 1.04^{18} = 2.0258$ millions.

   c) For continuous compounding the formula is $A(t) = Pe^{rt}$. Putting $P = 1, r = 0.08, t = 9$, $A(9) = 1(e^{0.08(9)} = e^{0.72} = 2.0544$ millions.

   Notice that the amounts are not that apart from each other, whether you compound once a year, twice, or continuously.

6. (10 points) Suppose the amount of ice in the Arctic ocean is decreasing at the rate of 5 percent per year. Write the equation for the amount of ice after t years if the initial amount is P cubic feet.

   Solution:

   This is same as compound interest formula except we subtract the rate of decrease.

$$A(t) = P(1 - r)^t = P(1 - 0.05)^t = P(0.95^t).$$

7. (15 points) Scientists can find the age of ancient artifacts by measuring percentage of Carbon-14 isotope. This is called carbon dating. Let N_0 be the original amount of Carbon-14 and $N(t)$ be the current amount, t years after the specimen died. k is rate of decay.

   a) If the equation for carbon decay is $N(t) = N_0e^{-kt}$, Solve for k in terms of t, N, and N_0.

   b) Half life is when $N(t)$ is half of N_0. Will half-life depend on what N_0 is? Why or why not?

   c) If half-life of Carbon-14 is 5730 years, when would $N(t)$ equal one quarter (25 percent) of N_0?

   Solution:

   a) Take logarithms of both sides after dividing by N_0.

$$\frac{N(t)}{N_0} = e^{-kt} \implies \ln(N/N_0) = -kt \implies k = -\frac{\ln(N/N_0)}{t}.$$

   b) Half life doesn't depend on N_0 or $N(t)$ because when you put $N(t) = N_0/2$ in the equation $N(t) = N_0e^{-kt}$ and then divide by N_0 it cancels out and you are left with just $1/2$. This is similar to how doubling time depends only on the rate, and not on intial amount.

c) N_0 becomes $N_0/2$ in 5730 years. Then $N_0/2$ becomes $N_0/4$ in another 5730 years, because as discussed above, it doesn't matter what you start with, it takes same amount of time to decrease by half. When you multiply $N_0/2$ by $1/2$ you get $N_0/4$. So totally it would take 11460 years to become one quarter of N_0.