Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 50 minutes. Total 100.

Please read the questions carefully before answering.

NO CELLPHONES.

1. (25 points) For the parabola given by $y = 2x^2 - 4x$ find the following:
   (a) *Equation of* the axis of symmetry.
   (b) The coordinates of the vertex.
   (c) The x-intercepts.
   (d) The y-intercepts.
   (e) The graph of the parabola with all of the above labelled clearly.

2. (10 points extra credit)

   a) In the parabola of problem 1, where is the focus located? Mark it in your graph and write down the coordinates.

   b) If the Profit (or loss when the value is negative) from a business is $2x^2 - 4x$ million dollars when x units are sold, what are all the values of x that give a profit? Note that this is the same expression as in problem 1.

3. (15 points) For the following sequence, find the common ratio, the formula for the n-th term and the value of the 100th term using the formula for the geometric sequence : (Leave your answers as powers).

$$\frac{1}{3},\ 1,\ 3,\ 9,\ 27, ..., \ldots$$

4. (20 points) Given that 1 million dollars are invested in a fund that returns 8 percent annually, find the following. You must use compound interest formula. Leave your answers as millions of dollars, for example 1 million and not 1,000,000.

   (a) Equation for the amount after t years.

   (b) The amount after 10 years.

   (c) The time it would takes to double. You must use logarithms.

   (d) The time it would take to quadruple. [Hint: Easy if you use the answer in (c)].

5. (15 points) The initial amount is 1 million, annual rate of interest is 8 percent, as above. You must use compound interest formulae.

   a) Find the amount after t years if interest is compounded twice a year.

   b) Find the amount after 9 years if interest is compounded twice a year.

   c) Find the amount after 9 years if interest is compounded continuously.

6. (10 points) Suppose the amount of ice in the Arctic ocean is decreasing at the rate of 5 percent per year. Write the equation for the amount of ice after t years if the initial amount is P cubic feet.

7. (15 points) Scientists can find the age of ancient artifacts by measuring percentage of Carbon-14 isotope. This is called carbon dating. Let N_0 be the original amount of Carbon-14 and $N(t)$ be the current amount, t years after the specimen died. k is rate of decay.

   a) If the equation for carbon decay is $N(t) = N_0 e^{-kt}$, Solve for k in terms of t, N, and N_0.

   b) Half life is when $N(t)$ is half of N_0. Will half-life depend on what N_0 is? Why or why not?

   c) If half-life of Carbon-14 is 5730 years, when would $N(t)$ equal one quarter (25 percent) of N_0?