Howard University Math Department

1. (10 points) Find the 20th term a_{20} of the following geometric sequence. You must use the formula for the n-th term a_n. You can leave answer as a fraction.

$$2, \frac{2}{3}, \frac{2}{9}, \frac{2}{27}, \ldots.$$

Solution:

$$a_n = ar^{n-1} \ ; \ a = 2, r = 1/3, n = 20. \ ; \quad a_{20} = 2\left(\frac{1}{3}\right)^{19} = \frac{2}{3^{19}}.$$

2. (Extra credit 10 points) Which of the following are polynomial functions, which are exponential functions? Explain.

$$3x^7 + 4x^3 + 1, \quad x^2 + \frac{1}{x^2}, \quad 2^x, \quad \frac{1}{2^x}$$

Solution: $3x^7 + 4x^3 + 1$ is polynomial because you are adding powers of x that have whole number exponents: $0, 1, 2, 3, \ldots.$

$x^2 + \frac{1}{x^2}$ is neither polynomial nor exponential because you have $1/x^2$ which is really x^{-2}, a negative power.

2^x is an exponential function – the variable is in the exponent.

$\frac{1}{2^x}$ is also exponential for the same reason, once you note that $\frac{1}{2^x} = \left(\frac{1}{2}\right)^x$.

3. (10 points) Using the compound interest formula find out how long it would take for \$ 1000 to double if the interest rate is 4 percent and interest is compounded annually. In other words, when does it become 2000 dollars?

You must use compound interest formula and logarithms.

Solution: The formula is $A(t) = P(1+r)^t$ where $A(t)$ is the amount after t years, P the initial amount and r the rate of interest. Here $P = 1000, r = 4/100 = 0.04$ and t is unknown.

$$1000(1 + 0.04)^t = 2000 \implies 1.04^t = 2 \implies t = \ln 2/\ln 1.04 = 17.67 \ years.$$

4. (10 points) Using the same rate of interest (4 percent) and initial amount (1000 dollars) find out how much will be in the account after 18 years if interest is compounded *continuously.* How long does it take to double ? You must use compound interest formula and logarithms.

Solution: The formula is $A(t) = Pe^{rt}$ where $A(t)$ is the amount after t years, P the initial amount and r the rate of interest. Here $P = 1000, r = 4/100 = 0.04$ and $t = 18$.

$$1000e^{0.04(18)} = 2054.43.$$

So it becomes slightly more than double.

Doubling time is given by

$$1000e^{0.04t} = 2000 \implies e^{.04t} = 2 \implies 0.04 \times t = \ln 2 \implies t = \ln 2/(0.04) = 17.33 \; years.$$