Howard University Math Department

1. (10 points) Find the vertex and axis of symmetry of the parabola $y = x^2 + 2x + 2$.

   Solution:

   Axis of symmetry is at $x = -b/2a = -2/2 = -1$. This also give x-coordinate of vertex.

   y-coordinate of vertex is obtained by putting $x = -1$ in equation: $y = (-1)^2 + 2(-1) + 2 = 1 - 2 + 2 = 1$.

   So vertex is at $(-1, 1)$.

2. (10 points) How far is the focus from the vertex in a parabolic dish of depth 2 feet and diameter of rim 4 feet?.

   Solution:

   Sitting the dish so its vertex is at $(0, 0)$ we get that the radius is 2 feet so for a point on the rim along the x axis the x-coordinate is 2.

   For that same point the depth is 2 means $y = 2$.

   Now we need to find a from $y = ax^2$.

   Plugging in $x = 2, y = 2$ we get $2 = a(4)$ which means $a = 1/2$.

   So the focus is $1/(4a) = 1/2$ feet or 6 inches from the vertex (bottom) of the dish along the axis.

3. (10 points) Check if the following are geometric sequences. Find common ratio if they are.

$$A. \quad 3, 9, 27, 81, ... \qquad B. \quad 4, 2, \frac{1}{2}, \frac{1}{4}, ...$$

   Solution: The first one is geometric. The common ratios is 3.

   The second one is not geometric. The ratio is 1/2 between 4 and 2, 1/2 and 1/4, but it is 1/4 between 2 and 1/2.