The first 2 problems involve the following data:

Annual fall temepratures in DC in 2020, 2010, 2000, 1990, 1980, 1970, 1960, and 1950:

62.1, 62.5, 58.2, 61.5, 61.9, 62.3, 59.5, 58.6

1. (15 points) Find the following:

   (a) average (b) median (c) mode.

   Solution:

   (a) There are 8 data points. So to find the average μ we need to add them and divide by 8.

$$\mu = \frac{62.1 + 62.5 + 58.2 + 61.5 + 61.9 + 62.3 + 59.5 + 58.6}{8} = 486.6/8 = 60.825.$$

   (b) median is average of 4th and 5th biggest items: (61.5+ 61.9)/2 = 61.7.

   (c) No mode because they all have same frequency.

2. (10 points) If the standard deviation σ of the above data is found to be 1.65, what percentage of the data are within two standard deviations of the average that you found in problem 1?

   NOTE: You DON'T NEED to calculate standard deviation. It is already given as 1.65.

   Solution: Twice the standard deviation is 3.3. Looking at the differences, we see that all of the data are within 3.3 of 60.825 or between 64.125 and 57.525. So 4 out of 8 are within two standard deviations. This makes it 100 percent of the data.

3. (Extra credit 10 points) Say whether true or false and justify your answer:

   a) Average temperatures in some years in the past were high, so global warming is not happening.

   b) The population of the world grows as a linear function of time.

   Solution:

   a) False. The overall trend over many years is that it has been getting higher.

   b) False. The population has been growing exponentially. Linear means it adds same amount of increase in every interval. That is not how it has been growing.

4. (20 points) (a) Say what kind of sequence is $1, \frac{3}{4}, \frac{9}{16}, \ldots$ and why.

   (b) Write the formula for the n-th term of the sequence

   (c) Find the sum upto 10 terms. You don't need to caclulate the whole thing, just simplify as much as you can.

   (d) What happens if you try to find the infinite sum?

   Solution: It is a geometric sequence with $r = 3/4$ and $a_n = 1 \times (3/4)^{n-1} = (3/4)^{n-1}$.

   (b) The sum is given by putting $r = 3/4, n = 10$ in the formula $a\left(\frac{r^n - 1}{r - 1}\right) : \frac{(3/4)^{10} - 1}{(3/4) - 1}$.

   The infinte sum converges because $r < 1$ and it equals $a/(1 - r) = 1/(1 - (3/4)) = 4$.

5. (15 points) Suppose installation of solar panels at a house cost $10,000$ dollars and every year they get back 800 in credits and reduction in electricity bills.

   Let the initial cost be -10000 and annual gain be $+800$. So the net cost after 1 year is $-10000+800=-9200$.

   (a) Write the net cost after 2 years, 3 years, and 4 years.

   (b) Say why the net cost sequence is an arithmetic sequence and write the net cost after n years in terms of n.

   c) When does the net cost equal zero? (This is the time it takes for the cost to be paid back).

   Solution:

   (a) The first 4 terms are $-9200, -8400, -7600, -6800$.

   (b) This is an arithmetic sequence because each time we are adding the same quantity 800.

   Using the formula for n-th term we get $a_n = -9200 + (n-1)(800) = -10000 + 800n$.

   (c) When this is zero we get $10000 = 800n$ and thus $n = 10000/800 = 12.5$

6. (10 points) Suppose you deposit 5000 dollars in a fund to save for the solar panels until you get $10,000$ dollars. if the fund pays 8 percent interest compounded per year, when would you have 10000?

   Solution:

   You want $5000(1.08)^t = 10000$. Dividing by 5000, $1.08^t = 2 \implies t = \ln 2/\ln(1.08) = 9$ years approximately.

7. (a) (10 points) In a Lotto, 3 different numbers are picked from 9, and the 9 numbers are also all different. It only matters what numbers are picked. Their order does not matter. How many possible picks are there?

   (b) (10 points) 1000 dollars are awarded to person who guesses all 3 numbers correctly. 500 dollars to person who guesses exactly 2 out of 3 correctly. 100 dollars to person who guessed exactly 1 out of 3 correctly. What is the average winning?

   Solution:

   (a) Number of ways to pick 3 out of 9 is $9P3/(3!) = (9 \times 8 \times 7)/6 = 84$.

   (b) There is only one right guess with all 3 correct. So $P(3) = 1/84$.

   There are 3 ways to pick 2 out of 3 correct numbers : $3C2 = 3$ (or you can say it is $3C1$ which is the number of ways to leave out 1). Then 6 ways to pick the incorrect one from the remaining 6. So $P(2) = 18/84$.

   There are similarly 3 ways to pick exactly one correct number. Then there are $6C2 = (6 \times 5)/(2 \times 1) = 15$ ways to pick 2 incorrect ones from the 6 remaining. $P(1) = 45/84$.

   Average or expected winning equals

$$1000 \times P(3) + 500 \times P(2) + 100 \times P(1) = \frac{1000}{84} + \frac{18 \times 500}{83} + \frac{45 \times 100}{84} = \frac{14500}{84} = 172.62\ dollars.$$