Please go to Update page and Course page to see information about class and to keep up to date. The links are main page of Canvas and also on http://nature-lover.net/math. You can see old notes from spring 25 etc at this website. It will help you prepare for class.

————————————————————————————————

Today: Counting methods: Permutations and Combinations

————————————————————————————————-

Quiz 8 on wed, Apr 15 : Calculating probability using basic definition; properties of probability, independent events and mutually exclusive events (multiplication and addition of probabilities, probability of opposite of an event or event not happening ; counting methods when repetition is allowed and not allowed, permutation and combination.

————————————————————————————————

Review:

Probability:

Probability is about using statistics to predict the chance of something happening.

So far we have learned:

In general, P(event) = number of favorable outcomes (frequency of event) divided by total number of outcomes.

P(sure event) = 1
P(impossible event) = 0
P(any event) is between 0 and 1 (includes 0 and 1).

P(opposite of an event or "NOT event") = 1 – P(event)
P(A and B) = P(A) x P(B) if A and B are independent events.
P(A or B) = P(A) + P(B) – P(A and B).
If A and B are mutually exclusive, P(A and B) = 0 and we get
P(A or B) = P(A) + P(B).

COUNTING METHODS

Basic idea: *Multiplication Principle*

If there are m ways to do thing A and n ways to do thing B, then there are mn ways to do both A and B (as long as A and B are "separate")

Example: If in an ice-cream shop there are 10 choices of flavors and 12 choice of toppings, there are 10 x 12 = 120 possible ways to choose a flavor and a topping. So probability that someone chose vanilla flavor and gummy bears topping is 1/120.

1. Repetitions allowed

Question: How many combinations of 3 digits each with 1,2,3,4, and 5 with repetitions allowed?

Each digit can be one of 5 numbers. So totally 5 x 5 x 5 = 125 possibilities.

Same problem with 10 digits:
Look at 2 digits first.
The possible combinations would be 11, 10, 12, 13,…., 20, 21, 22, …
Each digit can be one of 10 numbers 0,1,2,3,…,9 so total number of combinations would 10 x 10 = 100.
For 3 digits we will have 10 x 10 x 10 = 1000 combinations.

Problem 2: Suppose on three poll questions a student answered

randomly between choices A, B and C. How many different choices are possible? What is the probability that they got at least one wrong?
Answer:
3 choices each, they can be repeated, so 3 x 3 x 3 = 3^3 = 27 possibilities.

Probability of at least one wrong = 1 – Prob(getting all right)
because "getting at least one wrong" is opposite of "getting all right"
P(all right) = 1/27. So P(at least one wrong) = 1 – (1/27) = 26/27.

Question: how many outcomes from tossing 3 coins?

Each can be head or tail. HHH, HTH, TTH, etc.,
Total number = 2 x 2 x 2 = 2^3 = 8.

Probability of getting HHH is 1/8.

2. Repetitions not allowed, but order matters

Problem 3. Suppose a committee of 3 ranked members is chosen from 5. Say President, VP, and Secretary. How many possible?
Committee has to have 3 different people. No repetition! Assume they are all capable of each position.

President: 5 ways.
VP: 4 ways.
Secretary: 3 ways.
Totally: 5 x 4 x 3 = 60 ways.

This pattern is called **permutation**: Arranging 5 things in 3 places.
First place – 3 choices, and for each choice of first, have 2 choices for the remaining place.

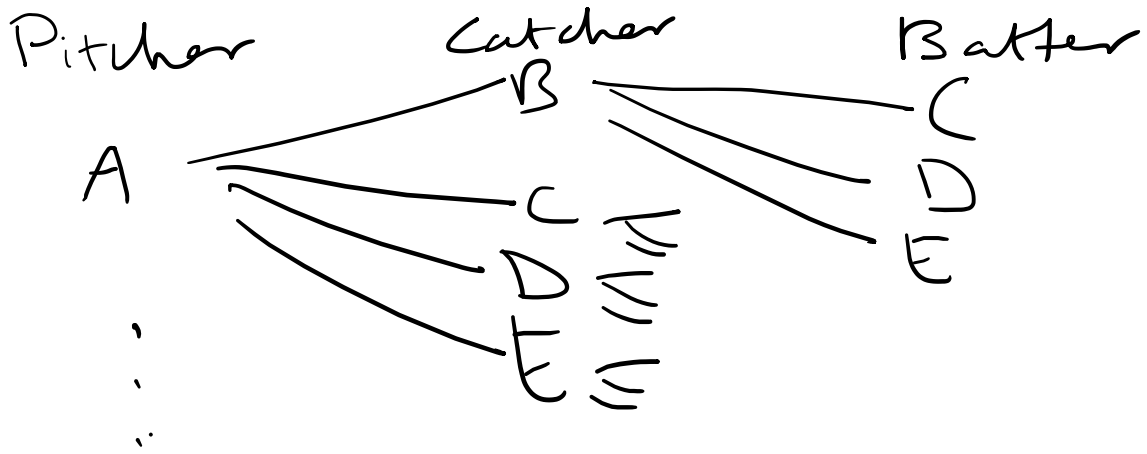
You and your best 4 friends are sitting and eating on one side of a table. Say they are A, B, C, D.
How many ways can you arrange all of you in 4 chairs?
First chair can have 5 people. Second chair must be a different person so it can be one of 4 people, and so on.
Can count it as 5 x 4 x 3 x 2 = 120.

In general 1 x 2 x 3 x…x (n-1) x n is denoted by n!
So 5! = 5 x 4 x 3 x 2 x 1 = 120.

5 x 4 x 3 = 5!/2!

So in general, nPk = number of permutations of n things taken k at a time is n! /(n-k)!. For example, 5P3 = 5!/(5-3)! = 5! / 2!

Example 1: How many ways are there to select a team with 3 positions -- catcher, a pitcher, and a batter from 5 baseball players-- if all of them can play all three of those positions?
Let A, B, C, D and E be the 5 players. We want to count the number of Ways to arrange 5 players in 3 positions.
Formula for nPk = number of ways to arrange n things in k places
= n! / (n-k)! = n (n-1)(n-2)…(n-k+1) (k numbers for k places)
n! means n(n-1)(n-2)…3x 2x 1
Earlier: number of ways to arrange 5 people in 5 places
= 5 x 4 x 3 x 2 x 1 = 120 = 5!



So total number of ways: 5 x 4 x 3 = 5!/2!

(Cancel because of multiplication) $= \frac{5 \times 4 \times 3 \times 2 \times 1}{2 \times 1} = 60$

3. Repetitions not allowed AND order doesn't matter

Suppose you just want to pick 3 people for the team, WITHOUT regard to position. We want to SELECT 3 people. ORDER DOESN'T MATTER.
We are NOT ARRANGING.
How many ways can we do that?
It would be smaller than 60. Why?
It will the same people, just that we don't count A, B, C differently from A, C, B and so on.
In fact, (A, B, C) , (A, C, B), (B, C, A), (B, A, C), (C, B, A) and (C, A, B) would all count as one!
So answer is 60/ 6 = 10 selections of 3 people from 5.
Reason: If we have a team of 3 people, they can be arranged in the 3 positions, pitcher, catcher and batter in 6 ways.
3 people in 3 places can be arranged in 6 ways.
Because 3P3 = 3 x 2 x 1 = 3! = 6.
So all the 60 teams can be arranged in 10 blocks of 6 teams, each with same 3 people.
COMBINATIONS = nCk = number of selections of k from n
= number of arrangements of k from n
Divided by number of arrangements of k from k

= nPk/k! = n!/(n-k)!k!

5C3 = 5! / (3! X 2!) = 120 / (6 x 2) = 10.

PROBLEM 1: What is the probability of picking correctly the final four teams out of 64 ?

(NOTE: 64! Is NOT equal to 60! times 4! In fact it is 60! Times 61 x 62 x 63 x 64).

First we need to find number of ways to SELECT 4 from 64.

Number of ways to arrange 4 out of 64 = 64P4 = 64 x 63 x 62 x 61

(Because 64!/(64-4)! = 64!/60! and you can cancel everything in numerator except the first 4).

So to get 64P4 you just multiply the first 4 numbers starting with 64. In general, nPk is the product of first k numbers starting with n)

Number of selections of 4 : = 64!/(60! X 4!) = 64P4 divided by 4! = 635, 376

Probability of winning = 1/635,376 because there is only one correct answer out of all the 635,376 possibilities.

Example 2 (for Wednesday) : From 52 cards with four suits clubs, hearts, diamond and spades, how many ways to **select** four spades?

Answer: There are 13 spades. Number of ways to select 4 is 13C4.

13C4 = 13 x 12 x 11 x 10 / (4 x 3 x 2 x 1) = 13 x 11 x 5 = 715.

Follow up question: Suppose you want to find P(4 spades).

Number of favorable outcomes = 715.

Total number of outcomes

= How many hands of 4 cards are possible out of 52?

= 52C4 = 52! /(48! x 4!) = 52 x 51 x 50 x 49 / (4 x 3 x 2 x 1)

P(4 spades) = 715/52C4 = 715/(26 x 17 x 25 x 49)

PRACTICE PROBLEMS FOR TODAY

1. How many numbers can you make for a lottery with 3 digits with all

digits different? Each digit can be one of the ten : 0, 1, 2, 3,..,up to 9. What is the probability of winning if you buy 10 tickets? For example it can be 123 or 109 or 854 etc. It cannot be 111 or 000 or 222 etc.

2. How many numbers in problem 1 if digits can be repeated? For example you can have 000 or 111 or 222….

3. How many ways to select a flavor and a topping from 10 flavors and 8 toppings ? What is the probability that a person chose chocolate flavor? What is the probability that they chose chocolate flavor and sprinkles for topping?

4. How many ways to select a forward, guard, and center from 10 basketball players? (Note that order matters here). What is the probability that a given player is a forward? What is the probability that players A and B are forward and guard respectively?

5. In problem 4 if we only choose 3 players regardless of position how many ways can it be done? What is the probability that A and B are in the team?