Please go to Update page and Course page to see information about class and to keep up to date. The links are main page of Canvas and also on http://nature-lover.net/math. You can see old notes from spring 25 etc at this website. It will help you prepare for class.

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Today: Probability and counting methods

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Review:

Probability:

Probability is about using statistics to predict the chance of something happening.

So for example, the average height of a US male is 5' 9". Standard deviation is 3 inches. So the chance that a random US male is of height between 63 and 75 inches (5'6" and 6'3" or two standard deviations from the average) is 95.4%.
This is because the Normal Distribution graph tells you the frequency of each value. For example, it is saying that 95.4 % of US men are between 63 and 75 inches. This is same as saying that probability of a random US male to be in that range is 95.4% or 0.954.

In general, probability of an event equals number of occurrences of that event divided by total number of events, or the percentage of occurrences of the given event. For example if in a group of 50 people, 20 of them speak English, then frequency of English speakers is 20 and probability that any random person picked from that group speaks English is 20/50 or 40% or 0.4.

Probability of an event
= Number of ways event can happen / Total number of events

For example, when a coin is tossed, P(H) = 1/2 or 50%.

P(sure event) = 1
Example: P(H or T) = 100% because it is either H or T.

P(impossible event) = 0
Example: P(neither H nor T) = 0.

P(any event) is between 0 and 1 (includes 0 and 1).

P(opposite of an event or "NOT event") = 1 – P(event)
Example: P(H) = 1-P(T) because getting H is opposite of getting T.
If you toss two coins, P(at least one H) = P(HH, HT, or TH) = 3/4.
This is same as P(NOT TT) = 1-P(TT) = 1- (1/4) = 3.4.

P(A and B) = P(A) x P(B) if A and B are independent events.
Example: What happens in first toss doesn't affect what happens in second toss. So P(TT) = P(T) x P(T) = (1/2) x (1/2) = 1/4.

P(A or B) = P(A) + P(B) – P(A and B).
If A and B are mutually exclusive, P(A and B) = 0 and we get
P(A or B) = P(A) + P(B).

Another example: On a throw of a dice, get 1, 2, 3, 4, 5, or 6.

Two dice: totally 6 x 6 = 36 different possibilities.

Calculate Prob(total < 10) = ?
For example, (1,1) results in a total of 2.
What are the favorable events? (1,1), (1,2)… Lots of possibilities.
Easier to calculate opposite.

The opposite of “total < 10” equals “total equals or greater than 10.”
The ways in which total $\geq$ 10 are (5,5), (5,6), (6,5), (6,6), (6,4), (4,6) or 6 ways.
P(total $\geq$ 10) = 6/36 or 1/6.
P(total < 10) = 1- (1/6) = 5/6 because getting < 10 is the opposite (or complement) of getting > or = 10.

Another way to do this:
P(total $\geq$ 10) = P(exactly 10) + P(exactly 11) + P(exactly 12)

This is always possible for Mutually exclusive events.

Mutually exclusive means they don’t happen together (intersection is empty). i.e, if you get exactly 10, you cannot exactly 11 or 12.

FACT: If A and B are mutually exclusive, **P(A or B) = P(A) + P(B)**

NOTE: In math, or set theory in particular, A or B means either A or B or both. Example: P(1 or H) means you get a 1 or a H or both.

If they are not mutually exclusive,
P(A or B) = P(A) + P(B) - P(A and B).

example: in a class of 10 there are 5 men, 5 women, 3 English majors of whom 2 are women. P(woman or English major) = P(woman) +P(English major) - P(female English major) = 5/10 + 3/10 - 2/10 = 6/10.
Count another way:
women or English major: 5 women + 1 male English major.

Independent events: when one doesn’t affect the other.
For independent events, **P(A and B) = P(A) x P(B) always.**

Question: one die: getting an odd number and getting < 5
1 and 3 are less than 5 AND odd. P(odd AND < 5) = 2/6 or 1/3.

The two events are: "getting an odd number" and "getting < 5."
You can multiply their probabilities to get 1/3. Their probabilities are 1/2 and 2/3 and the product is 1/3. But they are not really independent.

So if P(A and B) = P(A) x P(B) it doesn't mean A and B are independent.
But if they are independent, then definitely we will have :
P(A and B) = P(A) x P(B).

For example if the two events are "getting an odd number" and "getting < 4" then the probabilities are 1/2 and 1/2 but the intersection has probability 1/3 as before (1 and 3 are both odd AND < 4 and so it is 2/6 or 1/3). But product of 1/2 and 1/2 is 1/4, which is not equal to 1/3.

A die is cast and a coin is tossed. P(1 and H) = P(1) x P(H)

How many outcomes are possible: 6 x 2 = 12 outcomes.

example: P(odd and H) = 3/12 = 1/4. (1H or 3H or 5H are the favorable outcomes, 12 is the total number of outcomes).

P(odd and H) = P(odd) x P(H) = (1/2) x (1/2) = 1/4.

Question: throw of two dice: "getting a total of 5" AND "getting 1 on first toss" Are these independent?

total of 5: (1,4) , (2,3), (3,2), (4,1) – 4 ways.
getting 1 on first toss: (1,1), (1,2), (1,3), (1,4), (1,5), (1,6) -- 6 ways
Both together : (1,4) – just one way.
But when you multiply 4 and 6, you get 24.

Question: independent? No.
Two ways to see it:

1. If you get 1 on first toss, then to get total of 5 (at the same time!) then only possibility is (1,4)
2. When you multiply the number of favorable outcomes for each, you get 24, but number of favorable for both happening together is just 1.

Problem 2: Suppose on three poll questions a student answered randomly between choices A, B and C. How many different choices are possible? What is the probability that they got at least one wrong?
Answer:
3 choices each, they can be repeated, so 3 x 3 x 3 = 3^3 = 27 possibilities.

Probability of at least one wrong = 1 – Prob(getting all right)
because "getting at least one wrong" is opposite of "getting all right"
P(all right) = 1/27. So P(at least one wrong) = 1 – (1/27) = 26/27.

Another question: how many outcomes from tossing 3 coins?

Each can be head or tail. HHH, HTH, TTH, etc.,
Total number = 2 x 2 x 2 = 2^3 = 8.

Poll: Probability of getting HHH is 1/8.

First place – 3 choices, and for each choice of first, have 2 choices for the remaining place.

PRACTICE PROBLEMS FOR TODAY

1. A coin is tossed 3 times. Find the probability of getting three heads or no heads by adding the probability of each. Why can you add them? Then check your answer by counting number of ways to get 3 H or 0 H and dividing by total number of outcomes.

2. In problem 1, find probability of getting 3 heads by multiplying probability of heads in each toss. Why can you multiply them?

3. In a throw of 2 dice, find the probability of getting a total of less than 11 by finding the probability of the opposite.