Please go to Update page and Course page to see information about class and to keep up to date. The links are in main page of canvas and also on http://nature-lover.net/math. You can see old notes from spring 22 etc at this website. It will help you prepare for class.

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PLEASE TRY THE PRACTICE PROBLEMS AT END OF THESE NOTES!

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Today: sum of the terms of a geometric sequence.

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Review of geometric sequence: A sequence of real numbers where each term is multiplied by same number r to get the next term.

It goes like $a, ar, ar^2, \ldots, ar^{n-1} \ldots$ where $a_n = ar^{n-1}$

Today: Sum of a geometric sequence

Formula for adding n terms in a gometric sequence:

(For the moment, let the common ratio r be positive).

For sum of finite number of terms $a_1 + a_2 + \ldots + a_n$ denoted by $S(n)$: $$S(n) = a\frac{1-r^n}{1-r}.$$

Sum of an infinite number of terms: When $r < 1, S(\infty) = \dfrac{a}{1-r}$ because we can ignore r^n in the numerator because it goes to zero.

(You can keep adding more and more, but they will not exceed a/(1-r)).

When $r > 1$ the sequence grows to infinity or oscillates, and you cannot find the infinite sum.

Practice problem 1:

Sequence 1, 1/10, 1/100,1/1000,…… or 1, 0.1, 0.01, 0.001,…..

n-th term is $a_n = \dfrac{1}{10^{n-1}}$

Sum of first 5 terms: 1+0.1+0.01+0.001+0.0001 = 1.1111

Sum of first 10 terms: = 1.111111111

Sum using formula is $1\left(\dfrac{1-\frac{1}{10^{10}}}{1-\frac{1}{10}}\right) = 1.111111111$

We see that it approaches *the infinite, repeating decimal* 1.11111111….. which equals 10/9.

This is predicted by the formula for the infinite sum:

$$\text{Sum} = \frac{1}{1-r} = \frac{1}{1-\frac{1}{10}} = \frac{10}{9}$$

Practice problem 2: sum with r = 3, a = 1

Sum of first ten (= n) numbers

= 1 + 3+ 9+….+3^9 = $1 \times \dfrac{3^{10}-1}{3-1} = \dfrac{3^{10}-1}{2}$

Example Question 3: S(n) for 3,6,12,24,…

a = 3, r = 2, r-1 = 1, so S(n) = $\dfrac{a(r^n - 1)}{r - 1} = 3(2^n - 1)$.

You cannot use 1/(1-r) formula to find the sum for this because r is 3 and so r > 1 and sum goes to infinity.

NOTE: You can find the sum of a finite number of terms $S(n)$ using the same formula as above even if r is negative. For the infinite sum, if r is negative then we need that the absolute value of r given by $|r|$ to be less than 1 in order for sum to be finite. If r is $-3/2$ for example, its absolute value is 3/2 which is bigger than 1, and the sum will go to infinity as you add more and more numbers in the sequence.

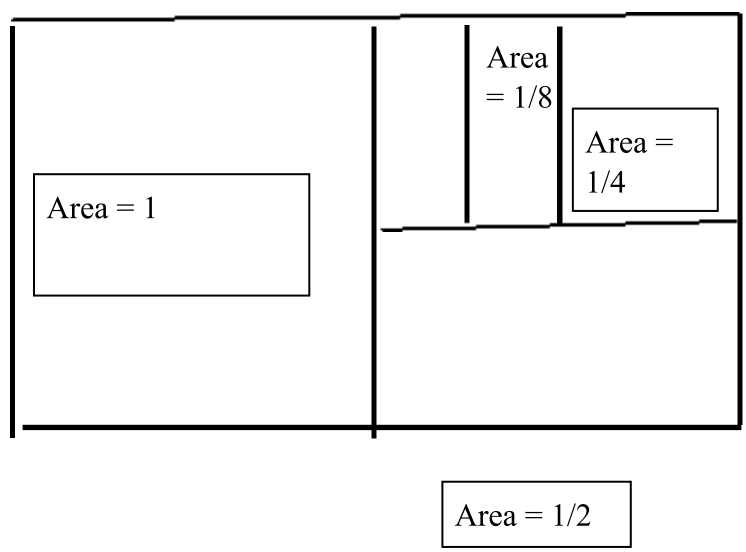
Infinite sum : Why is an infinite sum finite sometimes? See next page

(See picture below) We can **keep dividing into smaller rectangles forever**. Area of each rectangle is a term in geometric sequence 1, 1/2, 1/4, 1/8, …. The sum of their areas equals the total area, namely 2. So even though there are infinitely many of them, as we divide more and more, the new ones are so small that they don't add upto a lot.

a = area of first rectangle = 1. Each smaller rectangle has area 1/2, 1/4, 1/8 and so on.

Sum $= 1 + \frac{1}{2} + \frac{1}{4} + \ldots + \frac{1}{2^{n-1}} + \ldots = \frac{1}{1 - \frac{1}{2}} = 2.$

Total length of box is 2, total width is 1, so total area is 2.



Application to Investment, Loans etc.,

Investment : fixed amount each month:

Suppose m is the amount you deposit each month.

i = r/12 where r is annual interest rate (or rate of return on investment). Note: here r is NOT the common ratio.

How much money after n months?

The amount is calculated for each m using compound interest formula: $m(1+i)^k$ where n is number of months left.

So for example first deposit of m dollars collects interest for n-1 months, and so on until we get to last deposit of m which doesn't collect any interest.

$$A(1+i)^n = m(1+i)^{n-1} + \ldots. + m(1+i) + m$$

The amount on LHS is what one single deposit of A makes in n months.
The amount on the RHS is the amount from monthly deposits, and can be calculated using geometric sum formula.
a = m, r (common ratio, not interest) = 1+i,
so sum is

$$= m((1+i)^n - 1))/((1+i) - 1)$$

= $m((1+i)^n - 1))/i$

So we get
EQUATION: $A(1+i)^n = m((1+i)^n - 1))/i$

Supposing A were the loan on a car. The amount this will generate if invested is the amount on LHS.

The amount on the right is the total generated from monthly payments on the loan. The two need to be equal, because you are supposed to pay off the loan WITH interest.

PUNCHLINE: Solve for m from the EQUATION to get monthly payment on a loan of A : $m = A(1+i)^n/((1+i)^n - 1)/i)$

THIS IS THE FORMULA BANKS USE TO CALCULATE YOUR MONTHLY PAYMENTS.

For example, if A = 100,000, r = 6%, i = 6/12 = 0.5% = 0.005
Then m = 599.55 per month
Total amount paid = 215, 838.19

If r = 3%, we get m = 421.60, total amount paid = 151,776.

ASSIGNMENT 6 PROBLEMS FOR TODAY.
PLEASE PROVIDE STEP BY STEP EXPLANATIONS.

1. Find the sum of the first ten values of the geometric sequence 1, 2/3, 4/9, 6/27, …. Can you find the sum to infinity? If so, what is it?
2. Find the sum of the first ten values of the geometric sequence 1, 3/2, 9/4, 27/8, …. Can you find the sum to infinity? If so, what is it?
3. (Optional) Using the formula provided in these notes, find the monthly payment on a car loan with a period of 5 years (so 60 months) and annual interest rate of 3% (so monthly rate of 3/12 = 1/4 = 0.25%) if the car price was $30,000.