Please go to Update page and Course page to see information about class and to keep up to date. The links are in the Canvas main page and also on http://nature-lover.net/math. You can see old notes from spring 25 etc at this website. It will help you prepare for class.

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QUIZ 5 WED 3/4 ; TEST 2 WED 3/18 ; DETAILS ON UPDATE PAGE.

PLEASE FINISH ORAL PRESENTATIONS BY FRI 3/6.

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Today: more on continuous growth and decay, logarithms, and examples including half-life.

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Last time we saw how $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ approaches $Pe^{rt}$ as $n$ approaches infinity. It is like we are compounding interest so many times per year that it is like doing it continuously.

In general the function $Pe^{rt}$ models growth whenever it happens continuously.

Example showing how $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ approaches $Pe^{rt}$ :

You will see that if you increase $n$ in the first formula the values approach the value from the second formula.

If initial amount $P$ is 1000, $r$ is 0.1 and $t = 1$ year then after 1 year of continuous compounding the amount is $1000e^{0.1} = 1105.17$ dollars, using the second formula.

Using the regular (the first) formula and $n = 1$ (compounded once a year) it is 1000(1.1) = 1100. If $n = 2$ (compounded twice a year) it is 1000(1.05^2) = 1102.50. If $n = 10$ (compounded ten times a year) it is 1000(1.01^10) = 1104.62. If $n = 100$ (compounded 100 times a year) it is 1000(1.001^100) = 1105.12. If $n = 10$ (compounded ten times a year) it is 1105.16. So you see how they approach 1105.17.

Actually, any exponential function can be converted to this format, but the rate of growth will be changed.

For example, if $P$ is 1 and $r = 0.1$ then $A(t) = 1.1^t$ and the amount is compounded per year by 10%.

Now we can write $1.1 = e\hat{}k$ and solve for $k$ to get $k = \ln(1.1)$

So then $1.1^t = \left(e^{\ln(1.1)}\right)^t = e^{\ln(1.1)t} = e^{0.095t}$ ; Compare with $e^{rt} = e^{0.1t}$. You can see they are close. So for small values of $r$

$P(1+r)^t$ *and* $Pe^{rt}$ give about the same value. But remember that one is compounding once a year, the other is compounding continuously.

Review of properties of logarithm:

**Key point: logarithm of a number (to a given base) just the power of that number in that base.**

These properties are so useful that, for a long time, people used them to do big calculations (before calculators).

The properties come from the rule of exponents (or powers)

Power of a product = Sum of the powers
So logarithm of product = sum of the logarithms.

Similarly for division (results in subtraction of powers).

Power of a power = raise base to product of powers.

$$log_a(A^m) = mlog_a A$$

NOTE: $log_a(A^m) \neq (log_a A)^m$!

$ln(a^3)$ = 30 if $ln(a) = 10$ $because$ $ln(a^3) = 3ln(a)$

Question. $If\ ln2\ is\ A, then\ ln(8) =?$

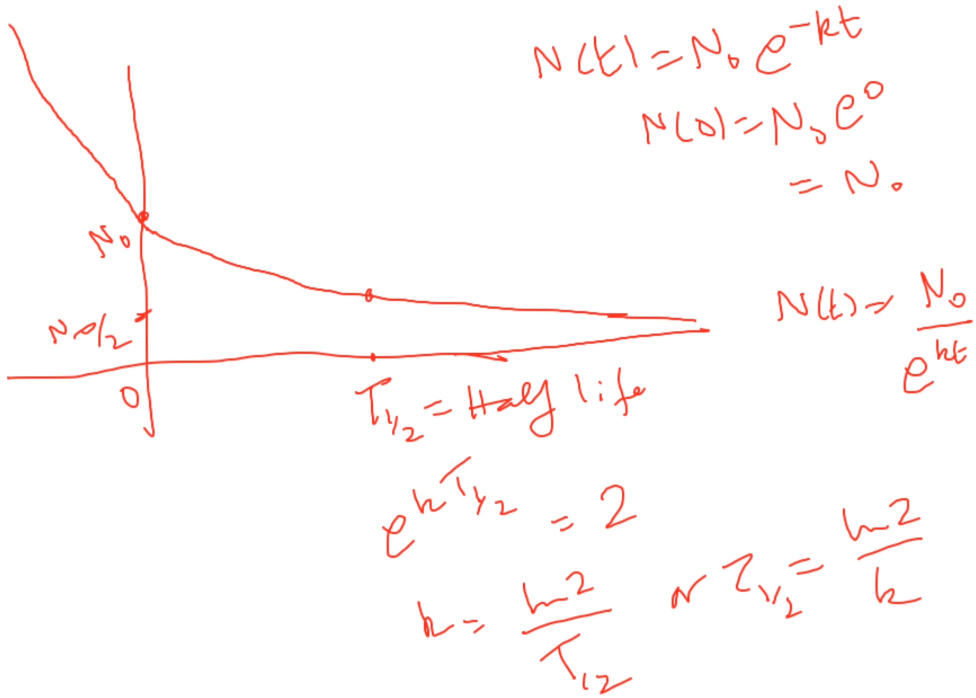
Answer: $ln8 = ln(2^3) = 3(ln2) = 3A$.

Half-life and carbon dating.

What is carbon dating?
Using radioactivity of carbon to measure age of different objects containing carbon, because the isotope Carbon-14 decays to Carbon-12 over time. If we know how much is left, we can figure out how long it took to get to that amount.

Function that measures amount of carbon looks like this:

$N(t) = N_0 e^{-kt}$ here $N(t)$ is amount after $t$ years, $N_0$ is initial amount, $k$ is a positive number that gives rate of decay (continuous decay).




Picture below shows how decay happens, and how to find half-life (called $T_{1/2}$ here). Half-life is similar to doubling time, it is the time taken for an amount to be reduced by half.
Climate change and carbon decay

https://yaleclimateconnections.org/2018/11/isotopes-point-to-the-culprit-behind-climate-change/

Practice problem: Half-life of carbon is 5600. 50% left now, how old is the bone? It is 5600 years old. If 25% is left, add another half-life to get 11200. So basically you can calculate how many times it got halved to find the age. Of course, if it is like 40% for example, then it is between one half and one quarter, so you will need to use the logarithms to find out how many times it halved, because when you halve 50% you get 25% so to get to 40% the number of times you multiply by 1/2 is between 1 and 2 and it can only be found using logarithm.

PRACTICE PROBLEMS FOR TODAY

1. 1 million dollars are invested in a fund. Calculate the amount if that investment fund returns 6% per year in 10 years if
   (a) it is compounded continuously
   (b) 12 times a year
   (c) 120 times a year
   (d) 360 times a year (CONTINUED NEXT PAGE)

2. Suppose the infections from a disease after first 4 days are given by 100, 110, 121, 133, respectively. One person tells you that it is growing (approximately) linearly. What would be the fixed amount added each day, approximately? How many people would be infected after 90 days if you believe their prediction? Another person tells you that it is growing (approximately) exponentially / geometrically. What would be the fixed percentage of growth each day, approximately? How many people would be infected after 90 days, under their prediction?

3. It is found that the amount of carbon-14 left in a piece of ancient wood is 6.125% of its original amount. If the half-life of carbon is 5730 years, how old is that piece of wood? Also write the exponential function $N_0 e^{kt}$ that would give you the amount after $t$ years, where $N_0$ is the initial amount (you don't need to find actual value, just leave it as $N_0$) and $k$ is the rate of decay. You can find $k$ using the same kind of equation as you use for doubling time, but this time you will use the half-life.