2/4/2026 Patterns in (environmental) math Class Notes

Today: Arithmetic sequences.

1. **QUIZ 3 Friday, on linear functions and arithmetic sequences**
2. **Test 1 next Wednesday. Details on update page.**

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Today: Linear functions review and arithmetic sequences

Application of linear functions:

Rental car charge, Electricity rates etc,

For example: A rental company charges 50$ per day and 25 cents per mile. Converting 25 cents to dollars we get 0.25. If $x$ is the number of miles driven in a day, and charge on that day is C(x) then C(x) = 50 + 0.25x.

An arithmetic sequence is the set of values of a linear function at the natural numbers: f(1), f(2), f(3),….

A sequence is a set of real numbers, ordered as

$a_1$, $a_2$ , $a_3$ , ……, $a_n$ , ….

A sequence is an example of a pattern among numbers

$a_n$ is called the $n$-th term.

0,1, 1, 2, 3, 5, 8, … is an example of a sequence. – Fibonacci sequence

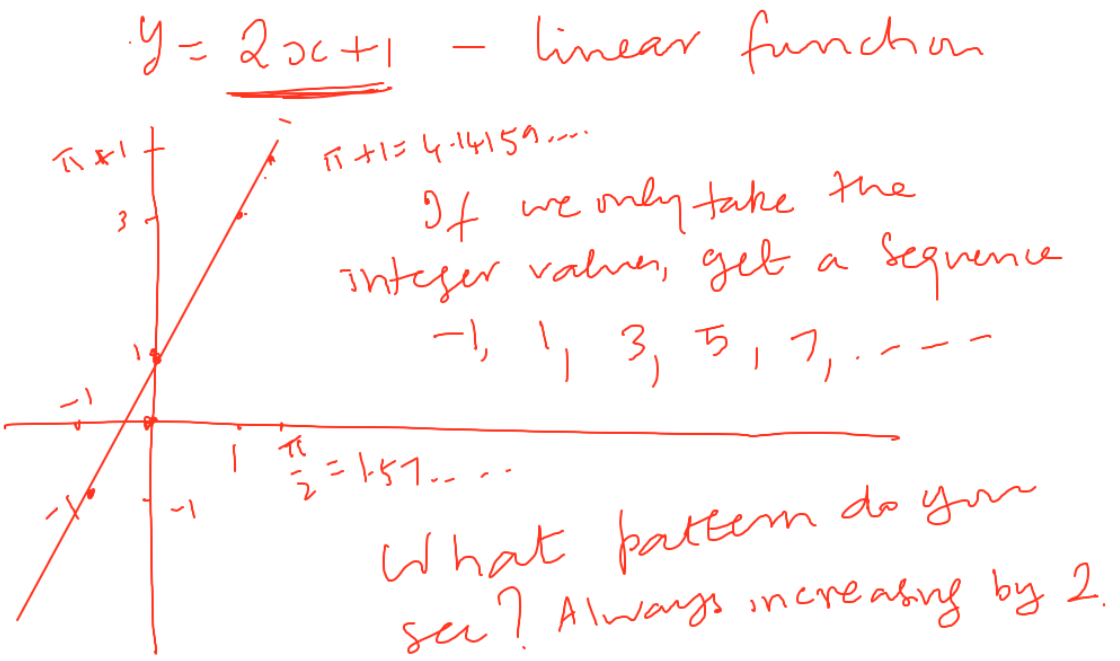
$a_n = \frac{u^n - v^n}{\sqrt{5}}$ , $u = \frac{1+\sqrt{5}}{2}$, $v = \frac{1-\sqrt{5}}{2}$, n = 0,1,2,…

Recall: For a linear function, During equal intervals of $x$ the $y$ value increases by the same amount. For example, any time $x$ is increased by 1, then $y$ is increased by the same amount, say $m$.

What is an arithmetic sequence?

Same as a linear function, except the inputs are only natural numbers, namely 1, 2, 3, … The output can be any real number.

For example, from the linear function $y = f(x) = 2x + 1$ we get the sequence $-1,1,3,5,7,\ldots$ by




looking at the values at the $x$ values $-1,0,1,2,3,\ldots$

Just as the linear function increases by 2 each time we increase $x$ by 1 (slope is 2), this sequence increases by 2 from one term to the next.

The slope becomes **common difference, denoted by d.** In this case, common difference is 2.

So an arithmetic sequence is a sequence of real numbers where each term differs from next by same amount d.

Question: arithmetic sequence or not?

-1, 3, 7, 11, 15,…

This is an arithmetic sequence with common difference 4.

What about 1, 3, 5, 7, 8, 9, 11, …?

It is not an arithmetic sequence!. Difference must be same every time!

1.5, 3, 4.5, 6, 7.5, … is an arithmetic sequence. Common difference is 1.5. Check: 6-4.5 = 1.5 and so on.

$\pi + 1, \pi + 3, \pi + 5, \ldots$ also an arithmetic sequence.

Common difference is 2.

FUN FACT: It has been proved recently that the sequence of prime numbers contains arbitrarily long segments that are part of arithmetic sequence.

Prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, …

For example: 3, 5, 7 are in a sequence with 3 terms

Similarly you can find arithmetic sequences of any length.

Is this an arithmetic sequence: 3.1, 6.2, 9.3, 12.4, 15.6, 18.7, ….

No, because 12.4 and 15.6 differ by 3.2 and not 3.1.

Chinese high speed rail:

2007 : 400 kms, 2010: 8400, 2013: 11000, 2016: 22000, 2019: 35000. Is this an arithmetic sequence?

How do you find the n-th term?

Let us take -1, 3, 7, 11, 15,…

1st term is $-1$. $a_1 = -1$.

2nd term is 3. $a_2$ = 3.

Similarly, $a_3 = 7, a_4 = 11, a_5 = 15$,...

Suppose you want to find the 10th term.

What is $a_{10}$? We would have added the common difference (= 4) 9 times.

So $a_{10} = -1 + (9 \times 4) = 35$.

So just like to get any point on the line $y = 2x + 1$ we just have to plug in the $x$ value, we can make a formula to get any term in an arithmetic sequence.

To get to the 10th term, added 9 common differences.

To get to any term, say $n$-th term we add 4 $n - 1$ times.

So $a_n = -1 + (4 \times (n-1)) = -1 + 4n - 4 = 4n - 5$

To find any term, just have to plug in the number of that term in this formula.

$4(1) - 5 = -1, 4(2) - 5 = 3, 4(3) - 5 = 7, \ldots a_{50} = 4(50) - 5 = 195.$

Find formula for n-th term from sequence.

The formula for a sequence starting with $a_1 = a$ and with a common difference of $d$ will be $a_n = a + (d \times (n-1))$

1.5, 3, 4.5, ….

$a = 1.5, d = 1.5$

So the $n$-th term will be $a_n = 1.5 + (1.5 \times (n-1))$

So for example $a_{50} = 1.5 + (49 \times 1.5)$

Question: 101st term of 1, 6, 11, 16, …

$a = 1, d = 5, a_{101} = 1 + (5 \times (101 - 1)) = 1 + 500 = 501.$

Suppose a sequence has $a_1 = 10$, and $a_{10} = 200$. Find the common difference and the formula for $a_n$.

Common difference (slope) = (200-10)/(10-1) = 190/9.
Alternatively, 10 + 9(d) = 200 => 9d = 190.
So $a_n$ = 10 + (n-1)(190/9).

PRACTICE QUESTIONS FROM TODAY.

1. Is $a_n = 2n + \pi^2$ an arithmetic sequence? If so, what is the common difference and what is the first term?
2. Find formula for n-th term $a_n$ of the sequence -5, -1, 3, 7, …. Then find the $100^{th}$ term $a_{100}$.
3. If in an arithmetic sequence $a_1 = 21$ and $a_{13} = 71$, find the whole sequence, including $a_n$.
4. A plumber charges $100 for materials and $50 per hour for labour. Write the charge C(t) as a function of time and find the charge if he works 3.5 hours.