2/4/2026 Patterns in (environmental) math Class Notes

Today: Arithmetic sequences and Applications.

Please go to Update page and Course page to see information about class and to keep up to date. The links are in the home page on Canvas and also on http://nature-lover.net/math

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1. **QUIZ 3 Friday, on linear functions and arithmetic sequences**
2. **Test 1 next Wednesday. Details on update page.**

Today: Sums of Arithmetic sequences.

The formula for the general term of a sequence starting with $a_1 = a$ and with a common difference of d will be $a_n = a + (d \times (n - 1))$

Question: an application problem

Number of petals in a flower are arranged in circles with 5, 8, 11, …flowers in circles 1,2,3,…

How many flowers in the 10th circle?

Answer:

$a = 5, d = 3, a_n = a + d(n - 1) = 5 + 3(n - 1) = 2 + 3n.$

10^{th} circle will have 2+3(10) = 32 petals.

Another example: Electricity rates, Rental car charges,…

Adding terms of an arithmetic sequence

From the example at the end this page (it is for natural numbers, but same idea works), we see that if we make a rectangle with $a_1 + a_n$ number of points in each of n columns, then the length of rectangle is $a_1 + a_n$ and the width is n, and area of triangle that is half the area of the rectangle is the sum. So the sum of the first n terms, namely, $a_1 + a_2 + \ldots + a_{n-1} + a_n = \frac{1}{2}(a_1 + a_n)n$

Here is the sum of 1, 2, 3, 4, with blue dots representing terms of the sequence and black dots representing the same sequence backwards. Total number of dots is twice the sum of 1, 2, 3, and 4. It equals (1+4) times 4 which is 20. So 1+2+3+4 is half of it, which is 10.

o o o o o (= 1+4)
o o o o o (= 2 + 3)
o o o o o (= 3+2)
o o o o o (= 4 + 1)

In other words,

Sum of first n terms = Half of n times (first + last).

Another way to see this: 1st + last = $a_1 + a_n = a_1 + (a_1 + d(n-1)) =$
$(2a_1) + d(n-1)$ will be the same if you add 2nd and 2nd from last, 3rd and 3rd from last, etc.,

For example, to add 1, 2, 3, …, upto a 100, write the sum in two ways and add:

Sum = 1 + 2 + 3 + …..+ 98 + 99 + 100
= 100 + 99 + 98 + …..+ 3 + 2 + 1

If you add the two, you get 101+101+…+101 (100 times). So it is just (1st + last) times number of terms. Then you divide by 2 because you added twice.

Example 1: Add -10, -3, 4, 11, 18, … upto n-th term: $d = 7$, $a_1 = -10$.

-10+(-3)+4+11+…..+(-10+7(n-1))
= n(-10 + -10+7(n-1))/2 = n(-20+7(n-1))/2.

Example 2: Adding first 100 terms of sequence -8, -4, 0, 4, ….
d = 4, a1 = -8.
-8 + -4 + 0+…+(-8 + (99)4)) = (100)(-8 + (-8+99(4))/2

= (100/2)(-16+(99)4)) = 50(-16+(99)4) = 50(380) = 1900.

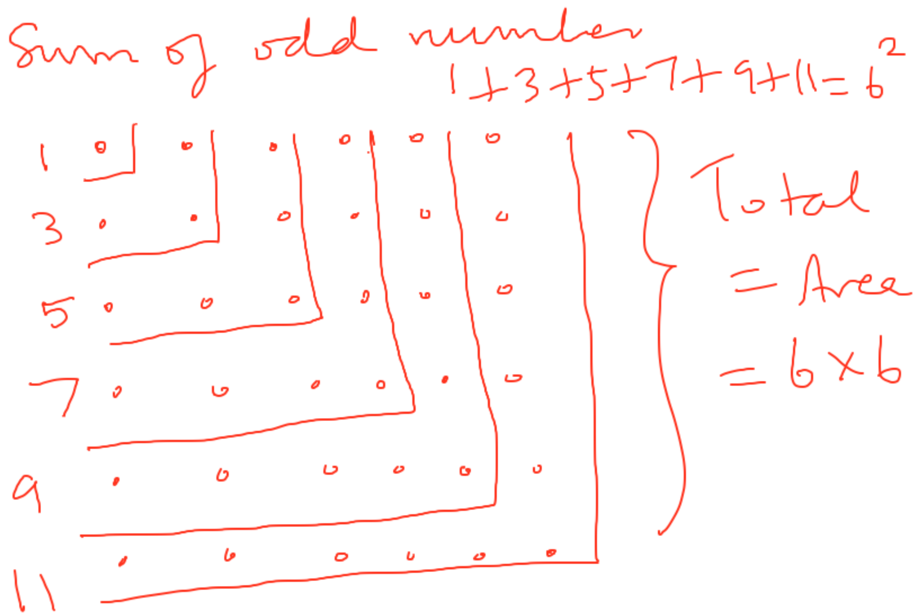
Nice formula for sum of odd numbers:

Suppose you have 1+3+5+…….+$(2n-1)$.

The sum is = $\frac{(1+(2n-1))(n)}{2}$

$$= (1 + 2n - 1)(n)/2 = (2n)(n)/2 = n^2$$

You can see this geometrically, in the picture below.



Another application example:

Suppose you start at a company at 45,000 $ salary. Each year they add 2500 $. How long before you make 100,000? How much would you have earned totally in that many years?

a_n = 100,000. What is n?

We want 100,000 = 45,000 + (2500(n-1)).

Solve for n to get n = 23.

Total amount you made equals the sum of 45000, 47500, 50000, …., up to 100000.

Using formula, this is (45000+100000)(23)/2 = 1,667,500.

Practice problems for today:

Suppose you start running 2 miles on first day, and each day you run for 0.25 miles more. How far would you be running on the 30th day?

How many miles would you have run for the whole month?

Answer should be 9.25 and 168.75 respectively. Show your steps and explain what you are doing.