Please go to Update page and Course page to see information about class and to keep up to date. The links are in canvas home page and also on http://nature-lover.net/math. You can see old notes from spring 25 etc at this website. It will help you prepare for class.

QUIZ 4 ON MAR 4. Extra credit 10 points!

STUDY GUIDE ON UPDATE PAGE SOON.

PLEASE BRING REGULAR CALCULATOR

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Oral presentation part 2 due by Mar 6.

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Review of geometric sequence: n-th term $a_n = ar^{n-1}$

Arithmetic sequence: add same number d (common difference) each time.

Geometric sequence: Multiply by same number each time. This number is called the *common ratio, denoted* r.*This letter is also used for rate of interest. Please make sure you know which it is, from the context.*

In general, please review:rules of exponents

$a^m a^n = a^{m+n}$, $a^m / a^n = a^{m-n}$, $(a^m)^n = a^{mn}$, $a^n b^n = (ab)^n$

Warning: *You can combine only when you have same base or the same exponent.*

APPLICATION OF EXPONENTIAL FUNCTIONS AND GEOMETRIC SEQUENCES

Geometric sequences and exponential functions:

Population growth:
Example: bacteria population growth happens by multiplication
Each bacteria divides into two.

If it doubles every hour, after t hours P(t) = P x 2^t, because after each hour, you multiply population by 2.
This is a geometric sequence. For each t = 1, 2, 3,.... (just natural numbers) we get a number.

If t can be any real number, then it is an exponential function.

(Kind of like how linear functions are related to arithmetic sequences:)

Exponential functions and polynomials.

In general, Exponential functions look like (a fixed number) times (a^x) where a is a fixed *positive number.*
Key point to note: THE VARIABLE IS IN THE EXPONENT

(Why only positive exponents? For example if a = -1 then (-1)^(1/2) is square root of -1 and it is not a real number).

Polynomial in general looks like $b_0 + b_1 x + b_2 x^2 + b_3 x^3 + \ldots + b_n x^n, b_i$ are called the coefficients and they can be any real number. But the exponents of x should only be non-negative integers or natural numbers. In other words,

n = 1,2,3,….

Linear functions of form $f(x) = mx + b$ and quadratic functions of the form $ax^2 + bx + c$ are all polynomials.

Basically power or polynomial functions have variable in the base and exponential functions have variable in the exponent.

Which are power or polynomial functions and which are exponential functions?

$x^{1.1}$, 1.1^x, π^2, $1/x^2$, x^x, $x^3 + 1.1^x$

$x^{1.1}$ is a power function but not a polynomial because exponent is not a non-negative integer.
1.1^x is an exponential function. Common ratio is 1.1.
$\pi^2 = \pi^2 x^0$ is a polynomial of degree zero.
$1/x^2 = x^{-2}$ is not a polynomial but a power function.
The last two are neither. x^x has neither base nor exponent fixed. The last one is a sum of an exponential and a polynomial function, so it is neither.

In general, any time growth happens in nature, it involves exponential functions.

Key point: growth happens by multiplication

Growing by r percent each year → Multiply by 1+r repeatedly

→ Multiply by $(1+r)^t$

APPLICATION OF EXPONENTIAL FUNCTIONS AND GEOMETRIC SEQUENCES

Example 1: Climate change and melting ice:

Suppose there was P amount of ice in 2000, and each year it is reduced by r percent.

Then after 1 year, there will be P(1-r) amount left, and after t years there will be $P(1-r)^t$. This is similar to compound interest formula except that instead increasing by r percent it is decreasing.

For example, if you had 1000 cubic meters and it is reduced by 5 percent then you will have 1000(1 - 0.5) = 1000(0.95) = 950 cubic meters. Next time, you reduce 950 (NOT 1000) by 5 percent. So you multiply 950 by 0.95 or 1000 by (.95)^2.

So after t years it will be $1000(.95^t)$.

Similar equation applies for depreciation of value of cars, houses, etc.

For example, in compound interest, if the amount after t years is denoted by $A(t)$, then $A(t+1) = P(1+r)^{t+1}$

$$= P(1+r)^t(1+r) = A(t)(1+r)$$

So each time we are multiplying by $1 + r$.
So for example, if rate is 5 percent, then you multiply by 1.05.

If interest is compounded more than once a year, say n times, then we need to **divide the rate by n** and **multiply the time by n**.

You get $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$

Reason:

Rate is annual rate. So it has to be divided by n to get the rate for the new period. For example if n is 2, then the period is 6 months, and rate of interest for 6 months would be r/2.

We multiply t by n because in t years you would have nt periods. This is the number of times interest is compounded in t years.

Example 2: Suppose you start with 1000 $. The interest rate is r.

$A(t) = 1000(1+r)^t$

How do you get initial amount from the formula?

Put t = 0 to get initial amount = 1000.

Suppose r = 10%. Then 1+r = 1.1.

How long it takes for amount to grow to 2000?

Set A(t) = 2000.

Get $2000 = 1000(1.1^t)$ => $2 = 1.1^t$ => $\ln 2 = \ln(1.1^t)$

=> $\ln 2 = t \ln(1.1)$ => t = 7.273 years.

So logarithm functions (log(x) or ln(x)) is the inverse of exponential functions. Whenever we want to solve for the exponent, we use logarithm.

The main property we used to solve for t is:

Logarithm of power = exponent times logarithm

So we had: $\ln(1.1^t) = t \ln(1.1)$.

PRACTICE QUESTIONS

1. Suppose in example 2 you start with 2000. When would it be 4000? If you start with 4000, when would it become 8000? What is the total time to go from 1000 to 8000?
2. In question 1, find the time it takes to go from 1000 to 8000. Do you get the same result? Can you explain why?
3. Repeat example 2 with interest being compounded monthly. In other words put n = 12 in the formula $A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$ and find amount after 7.273 years. Can you guess why the answer is close to what we got in example 2?
4. Power function or polynomial or exponential function?

$$x^{-2}, 3x^2 + 4x, 1.0001^x$$