2/2/26 Patterns in (environmental) math Class Notes

Today: Graphs of linear functions.

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1. **QUIZZES ARE 30% OF GRADE. IF YOU PAY ATTENTION, EASY TO GET RIGHT ANSWER.**
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QUIZ 3 FRI FEB 6. TEST I WED FEB 11. MORE DETAILS SOON.

First we looked at some pioneers of environmental movement among minorities (see notes from fall 2019, 1/14, titled Introduction to Climate Change, page 4. It is linked on update for today on Update page). We also read about problems facing the environment, etc.,

Linear functions and their graphs

How to tell whether a function is linear.

What is a linear function?

A function whose **rate of change is constant**.

During equal intervals of x the y value increases by the same amount.

For example, any time x is increased by 1, then y is increased by the same amount, say m.

This value m is called the slope of the line.

You can tell whether the graph will go down or up based on whether m is negative or positive.

Say the value when $x = 0$ is $y = b$.

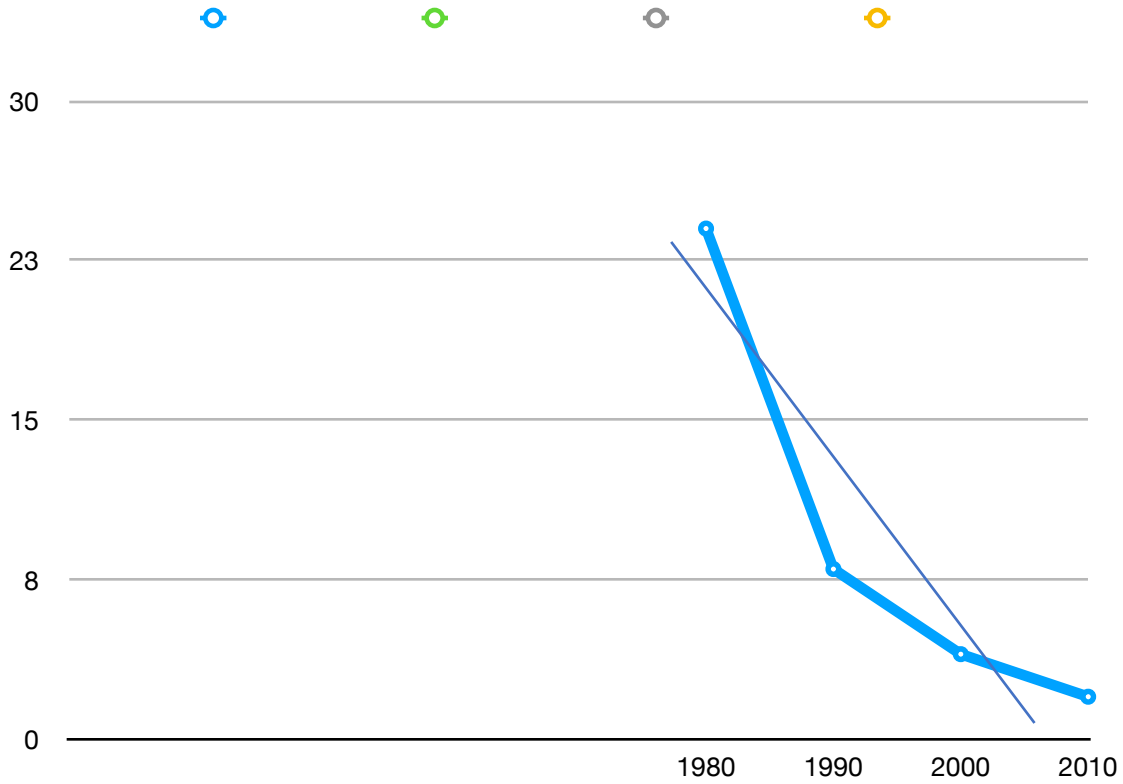
This is the y intercept.

So $y = f(x) = b + mx = mx + b$

The slope or rate of change can be found between any two points $(x_1, y_1) and (x_2, y_2) = \frac{y_2 - y_1}{x_2 - x_1}$.

(There is a question involving a graph similar to the following, in assignment 3. See below)

The chart above shows the prices of solar panels over the years:



The first number (to be on x-axis) is the year and the second number (to be on y-axis) is the price per Watt in 2012 dollars: (approximately)(1980, 24), (1990, 8), (2000, 4), (2010, 2).

The thin blue line is (approximately) a line that “fits” the data. It is roughly the line whose average distance from the points on the graph of the data is the smallest. Excel and other programs have functions that do this.

The chart shows that the prices are not decreasing in a linear fashion.

But we can also do this without the graph:

Compare the slopes between different pairs of points.

Say (1980, 24) and (1990, 8) are chosen.

The slope of line joining them is

$$(8-24)/(1990-1980) = -16/10 = -1.6$$

But if we take (1980,24) and (2000, 4) then the slope of line joining them is

$$(4-24)/(2000-1980) = -20/20 = -1$$

So the slope is becoming less negative, which means prices are still decreasing, but at a slower rate.

Possible exercise for you (not part of any assignment): Find some data from your topic about a particular item, say for example the amount of tree cover in a city and see whether the change in that amount has been happening in a linear fashion. Also try to make a graph.

Questions:

To find slope of any line, enough to find slope between any 2 points on the line?

What is the relation between slopes and whether a function is increasing or decreasing?

Yes. If it is a straight line, then the slopes will be the same no matter which two points you choose.

Review of polynomial functions:

A polynomial function is basically a sum of powers of x multiplied by real numbers. These numbers are called the coefficients.

So for a linear function $y = f(x) = mx + b$ the coefficients are $m \; and \; b$. m is slope and b is the y-intercept.

In general a polynomial function looks like

$$y = a_0 + a_1x + a_2x^2 + a_3x^3 + \ldots . + a_{n-1}x^{n-1} + a_nx^n$$

The powers should be whole numbers such as 0, 1, 2, 3, 4, …

f(x) = k would be a polynomial of degree 0.

$f(x) = ax^2 + bx + c$ is a general quadratic function, and it is of degree 2.

Degree is the highest power of x in the polynomial expression.

$f(x) = \pi^2$ is just degree 0 because π^2 is just a number.

$\pi = 3.14159 \ldots .$ is an irrational, even transcendental (not algebraic) number.

But it is just a fixed quantity.

$\sqrt{2}$ is an example of an algebraic number that is irrational. It is algebraic because it is the solution of a polynomial equation $x^2 - 2 = 0$. (The other solution of this equation is $-\sqrt{2}$).

Practice questions from today:

The questions are based on data about the growth of High Speed Rail network in China (watch the video linked on the update page if you want to learn more about it). High speed rail helps reduce carbon emissions by reducing car and plane travel.

In 2007, China had a total of about 400 kms in length of high speed rail tracks. This gives a data point (2007, 400).

Here are some other data points: (From Wikipedia, rounded up)

(2010, 8400), (2013, 11000), (2016, 22000), (2019, 35000), (2022, 42000), (2024, 48000).

1. Draw a graph plotting these points and join them by straight lines.
2. Find the slopes of the line segments between the points.
3. When is it increasing and when is it decreasing? Is the rate of increase slowing down?
4. Estimate the length in 2028.