Please go to Update page and Course page to see information about class and to keep up to date. The links are in canvas course home page and also on http://nature-lover.net/math. You can see old notes from spring 25 etc at this website. It will help you prepare for class.

QUIZ 4 ON FEB 20. STUDY QUADRATIC EQUATION, PARABOLA, AND GEOMETRIC SEQUENCE

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APPLICATION OF PARABOLAS AND QUADRATIC EQUATION

AREA OF A RECTANGULAR FIELD OF SOLAR PANELS

Practice problem: Suppose 2000 feet of fencing available.

Write down area as a function of width. l+w = 1000 => l = 1000-w

Area = w(1000-w) = 1000w -w^2. Vertex would be at -b/2a.

Question: Find maximum

Maximum area if perimeter is 2000 happens when width is 500

because b = 1000 and a = -1 and so -b/2a = 500.

So when width is 500, area is max. Area = value of A at w = 500

and that is 500(1000-500) = 250000 sq. ft.

Other examples:

MILEAGE IN A CAR AS A FUNCTION OF SPEED

$m(s) = -0.018s^2 + 1.93s - 25.34$

Best mileage happens when $s = -1.93/2(-0.018) = 53.6mph$

MOTION OF AN OBJECT

$$d(t) = -\frac{1}{2}gt^2 + v_0t + d_0$$

Can find the time at which it reaches maximum height, using this.

g is the acceleration, v_0 is initial velocity, d_0 is initial position.

Geometric sequences and exponential functions:

Geometric sequence: **each time we will multiply by same number**.

The common multiplier is called **common ratio.**

Which of these are arithmetic, which are geometric, and which are neither?

1, 7, 13, 20,… Arithmetic, d = 6.

2, 4, 6, 8,… Arithmetic, d = 2.

2, 4, 8, 16,… Geometric, r = 2 (common ratio)

1/2, 1/4, 1/8, 1/16,… Geometric, r = ½

Note: Geometric sequence increases when r > 1 and decreases when 0 < r < 1. If r = 0 or 1, get a constant sequence, like 1,0,0,0,…or 1,1,1,1,….

What if r = -1 or some negative number?

Get an alternating or oscillating sequence like 2, -4, 8, -16,….

If production of solar panels grows by 42% or 0.42 each time, then each time you multiply by 1.42.

So after two years you will have 20 x 1.42 x 1.42 = 20 x (1.42^2)

Here rate of growth annually is 42%.

Example: compound interest formula.

Amount every year = Amount in previous year times a fixed factor.

Formula: $A(t) = P(1 + r)^t$

Here P is initial amount, r is annual rate of interest and t is number years.

So for example, if you start with 1000$ and rate of interest is 5%

then after 5 years, you will have $1000 \times 1.05^5 = 1276.28$ dollars after 5 years. Basically you are multiplying by 1.05 each year.

Example 1: compound interest formula.

Annual interest is 10%, initial amount is P.

After t years you will have $A(t) = P(1.1^t)$ because 10% is 0.1 and so $1 + r = 1.1$.

Where can we see them in nature and in real life?
Population growth:
Example 2: bacteria population growth happens by multiplication
Each bacteria divides into two.
If it doubles every hour, after t hours P(t) = P x 2^t
This is a geometric sequence. For each t = 1, 2, 3,.... (just natural numbers) we get a number.

If t can be any real number, then it is an exponential function.

(Kind of like how linear functions are related to arithmetic sequences:)
In general, any time growth happens in nature, it involves exponential functions.

Testing whether something is geometric:
Is the following a geometric sequence? If not, why?
3, 1/3, 1/9, 1/81, ...

Clearly not an arithmetic sequence. Differences are different.
Not a geometric sequence!
Not multiplying by same number! No common ratio.
If you start with any real number, say a,
and multiply by SAME number, say r,
get a geometric sequence whose n-th term $a_n = ar^{n-1}$
one way to find the common ratio r:
Divide any term by previous term. So in above sequence, 1/3 divided by 3 is 1/9 but 1/9 divided by 1/3 is 1/3 and the ratios are not same.

Example 3: Find common ratio of the following, if it is geometric.

3, 12, 48, 96,…. Not a geometric sequence because even though first three terms are in a geometric sequence with r = 4, the fourth is not. 48 times 4 is not 96.

Example 4: 1, 10, 1000, 10000, … is a geometric sequence with r = 10. So the n-th term would be 1 times $10^{(n-1)} = 10^{(n-1)}$. (That is 10 raised to the power of n-1).

PRACTICE QUESTIONS FROM TODAY

Please provide step by step answers,
With explanations

1. If the number of people infected in a state by a virus increases by 5% each day, and initially 100 people were infected, write an equation for the number of people infected after n days. It must be a geometric sequence of the form $I_n = Ar^{n-1}$ where A is the first term and r is the common ratio.

2. Find out how many people are infected after 10, 20 and 30 days, and how many are infected after 300 days.

3. Repeat question 2 when the number of people infected increases by 6% each day and compare them with the numbers in question 2.