

Please go to Update page and Course page to see information about class and to keep up to date. The links are in the home-page on Canvas and also on <http://nature-lover.net/math>.

TEST 1 ON FEB 13 FRIDAY. MORE ON UPDATE PAGE.

Today: Quadratic functions and applications

Review of sum of arithmetic sequence.

Formula for adding

$$a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n = \frac{1}{2}(n)(a_1 + a_n)$$

Examples:

Formula: Sum = Half of number of terms times (first + last)

$$1 + 2 + \dots + n = \frac{1}{2}(n)(n + 1)$$

So $1+2+\dots+100 = (50)(101) = 5050$.

$1+3+5+\dots+a_n$ (sum of first n odd numbers) $= (1/2)(n)(1 + a_n)$

Now $a_n = 1+(n-1)2 = 1 + 2n-2 = 2n-1$ is the formula for n -th odd number.

$$= (1/2)((n)(1+(1+(n-1)2))) = (1/2)(n)(2+2n-2) = n^2$$

Example: $1+3+5+7 = (1/2)(4)(1+7) = 2(8) = 16$.

Suppose you want to know how many are there from 1 to 7.

$a = 1$, $d = 2$ (common difference)

You can find it by saying

$$7 = 1 + 2(n-1) \Rightarrow 2(n-1) = 6 \Rightarrow n-1 = 3 \Rightarrow n = 4.$$

PARABOLAS AND QUADRATIC EQUATIONS

Examples of parabolic objects

Path of any falling object, including some comets.

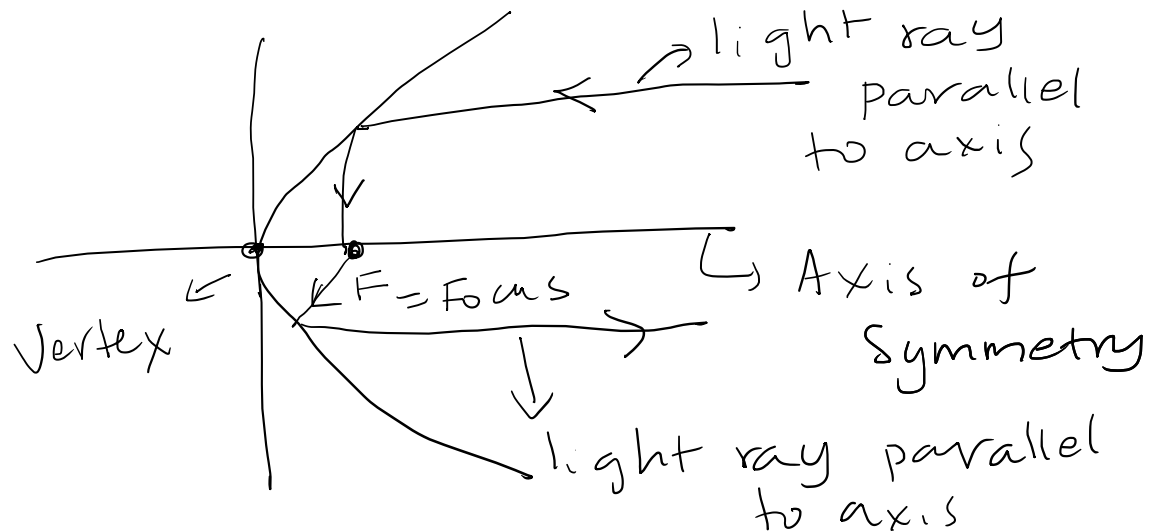
(Check out “Don’t look up” movie).

(In general the path of celestial objects could be a circle, ellipse, hyperbola or parabola...all curves described by quadratic equations — equations with x^2 in them, as opposed to linear equation which is of form $y = mx + b$).

Satellite dish, some telescopes, headlights, parabolic microphones, concentrated solar power generators,.....

Why do all of these instruments use the parabolic shape?

Reason is that the parabola has this nice property that all lines parallel to its axis get reflected through a single point called the **focus** of the parabola. So whether it is light from a distant star, or a signal from a satellite, or sounds from the middle of a football field, they get focused at one point and hence magnified. Same thing happens if light is placed at the focus of the parabolic mirror of a headlight. It gets reflected parallel to the focus, so you get a concentrated beam of light.



Focus and vertex and axis of symmetry.

Usually, formula of a parabola is given as $y = ax^2$

Example: $y = 2x^2$ then axis of symmetry is y-axis, vertex is at (0,0),

$a = 2$, focus is at $1/(4a) = 1/8$ distance from vertex along the axis.

(If equation is given as $x^2 = 4by$ then b is the focus distance from the vertex, because this is really same as $y = \left(\frac{1}{4b}\right)x^2$

and so $a = 1/(4b)$ and $\frac{1}{4a} = b$).

Question: finding focus and vertex

$y = 4x^2 \Rightarrow a = 4, 1/(4a) = 1/16$.

So the focus is at a distance of $1/16$ from the vertex.

Quadratic equation in general: (example equation of ellipse, etc)

General quadratic equation

$$ax^2 + by^2 + cxy + dx + ey + f = 0$$

Parabola: $y = ax^2$ (Put $c = -1, b, c, d, f$ all 0)

Circle: $ax^2 + ay^2 + dx + ey + f = 0$
 e.g. $a = 1, b = d = e = 0, f = -1$, get $x^2 + y^2 = 1$

hyperbola: $ax^2 - by^2 = 1$ a, b both > 0

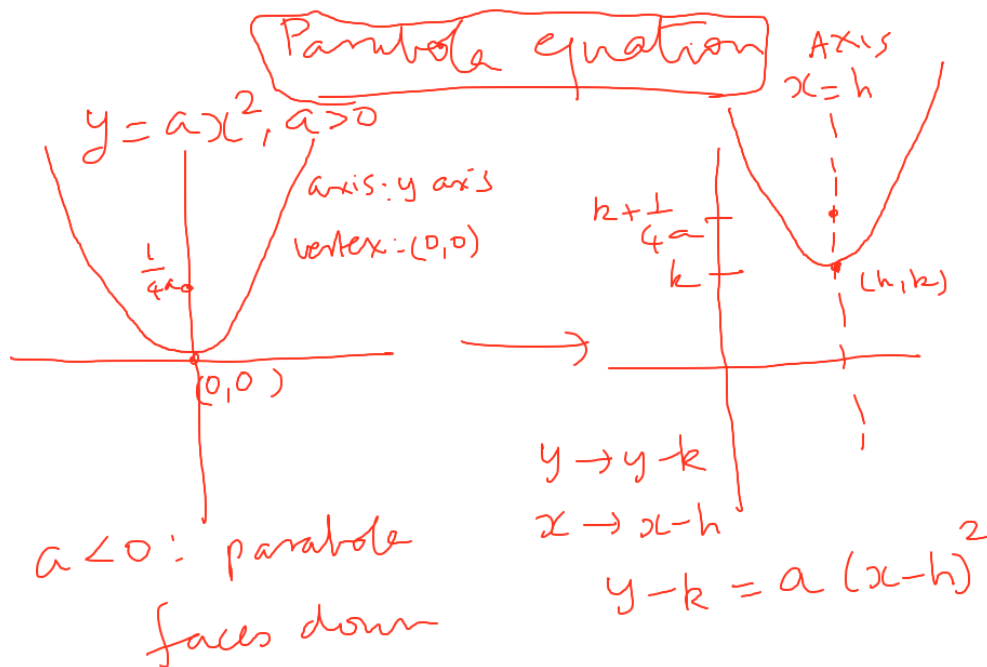
ellipse: $ax^2 + by^2 = 1$ a, b both > 0

Quadratic equation's graph could be a circle, ellipse, hyperbola, parabola, or even a pair of lines.

For example $xy = 0$ represents points that are either on x -axis which is $y = 0$ or on y -axis which is $x = 0$.

For this course, we will only look at equations of parabolas with axis of symmetry vertical (parallel to y -axis). If axis is horizontal the equation will look like $x = ay^2$ but we won't be dealing with those.

Parabola with vertex at (h,k) and axis of symmetry $x = h$.



Equation would become $y - k = a(x - h)^2$.

We are just replacing $y \rightarrow y - k$ and $x \rightarrow x - h$.

Practice problem: Write equation of parabola with vertex $(1, -2)$ and axis vertical. Leave a as it is without a specific value.

Put $h = 1, k = -2$ in the formula to get $y - (-2) = a(x - 1)^2 \Rightarrow y + 2 = a(x - 1)^2$.

Example 1: Find equation of parabola with vertex $(1,0)$ and axis vertical.

$$y - 0 = a(x - 1)^2 \Rightarrow y = a(x - 1)^2.$$

Getting vertex and axis of symmetry and focus from quadratic equation.

Suppose we are given $y = 3x^2 + 4x + 5$.

$a = 3$ because it is always the coefficient of x^2 . So we know it is facing up and the axis is vertical.

Can find (h, k) or the vertex we can complete the square

and it will turn out that $h = -b/2a$ where b is the coefficient of x .

Note: $-b/2a$ is just the first half of the quadratic formula! In fact, if you set the right hand side equal to zero, you are getting the points where $y = 0$ and those are just the points where the graph cuts the x -axis. Remember that the quadratic equation $ax^2 + bx + c = 0$ has one, two, or no solutions depending on whether $\sqrt{(b^2 - 4ac)}$ is zero, nonzero, or imaginary.

Here $b = 4$ and $h = -4/(2 \times 3) = -4/6$.

So we know it looks like $y - k = 3(x - (-4/6))^2$.

The value of k will be the value of y when $x = -4/6$.

Plug in $-4/6 = -2/3$ in $y = 3x^2 + 4x + 5$ and you get $k = 11/3$. So the vertex is at $(-2/3, 11/3)$ and you can see it is above the x -axis, so parabola cannot cut the x -axis.

Axis of symmetry is the line $x = h$ or in this case, $x = -2/3$.

Practice problem: $y = x^2 + 2x + 1$

What are a , b , and c ? What is (h, k) , the vertex? Axis of symmetry? $h = -b/2a = (-2)/(2 \times 1) = -1$.

Axis of symmetry is $x = -1$, $(h, k) = (-1, 0)$ because when you plug in -1 into the equation you get $y = 0$.

NOTE: The x coordinate of the vertex is h and the y coordinate is k . You get the y coordinate by plugging in the x coordinate in the equation of the parabola.

Example 2: Getting vertex and axis of symmetry.

$$y = 2x^2 + 4x \Rightarrow h = -b/2a = -4/(2 \times 2) = -1, k = f(-1) = 2(-1)^2 + 4(-1) = -2.$$

ASSIGNMENT 4 PROBLEMS FROM TODAY

1. Which of the following are of parabolic shape?

Satellite dish, Dish based concentrated solar power (CSP) generator, trough shaped CSP, Orbit of earth, Basketball, Football, St. Louis Arch, Bay bridge cables, path of a baseball thrown at an angle, orbit of a satellite.

2. A satellite dish is in the shape of a parabola $y = ax^2$. If the diameter of the dish at the edge is 2 meters and the depth

is $1/4$ of a meter, (so $x = 1$ when $y = 1/4$), find the location of the focus.

3. Find the equation of parabola with vertex at $(1,1)$ and axis vertical.
4. Find the equation of the axis and the location of the vertex and the focus of the parabola given by :

$$y = x^2 + 4x + 1$$