Midterm Exam   Howard University Math Department   Sitaraman

PLEASE PROVIDE STEP BY STEP SOLUTIONS. Total 100.

Scan answers into SINGLE PDF file. Then click on "Create Journal Entry," then write "<your name> Patterns final" in title and use Attach file button to submit exam

1. (10 points) Name two sources of greenhouse gas emission other than automobiles.

Answer: Other sources: power plants, agriculture, buildings, factories…

2. (20 points) For the following sequences, find the formula for the n-th term an and the value of the fifth term using the formulae for the arithmetic sequence or geometric sequence depending on what it is:

(a)   0.1, 0.01, 0.001,….   (b) 1, 15, 29, . . .

(a) is geometric.

 Common ratio r = 0.1. For example, 0.1 times 0.1 is 0.01.

Fifth term is ar^4 = (0.1)(0.1)^4 = 0.1^5 = 0.00001.

(b) is arithmetic. Common difference is d = 14.

$a_5 = 1 + (4)(14) = 57.$

3. (10 points) Find the axis of symmetry and vertex of the parabola given by the equation $y = x^2 - x$.

(10 points extra credit) What is the property of the parabola that makes it suitable for solar thermal systems and satellite dishes alike?

Answer: Axis of symmetry is $x = -b/2a = -(-1)/2(1) = ½$.

To find location of vertex ON the axis of symmetry, put $x = ½$ in the equation: $y = (1/2)^2 - (1/2) = ¼ - ½ = -1/4$.

So vertex is located at (1/2, -1/4)

The main property of parabolic mirror or dish is that light rays parallel to axis of symmetry get reflected (focused) on the focus and light from focus gets reflected parallel to axis of symmetry.

4. (20 points) The amount in a certain investment grows according to the formula $A(t) = 1000 \times 1.06^t$.

a) What is the annual rate of interest?

b) Write the equation to find the time it takes to grow to 3000. You can leave the answer in terms of logarithms.

c) Write the equation to find amount after 10 years.

d) How much was there at the beginning?

Answers:

a) annual rate of interest (growth rate) = 0.06 or 6%

b) To grow to 3000 we set $3000 = 1000(1.06^t)$ and then solve for t. This can be done by first dividing by 1000 and then applying logarithm to both sides. We get $3 = 1.06^t$ and thus $\ln 3 = t$ times $\ln(1.06)$.

Final answer is t = ln3/ln(1.06).

5. (20 points) In a certain town 30 solar panels are added to homes every year. So after one year, 30 panels, after two years, 60 panels, and so on. How many solar panels would there be in the town after 20 years? Use the formulae for finding the n-th term and sum of an arithmetic sequence.

It is easy to see that n-th term is just n times 30. You can also get this as a_n = 30 + (n-1)30 = 30n. So after 20 years you have 30 times 20 or 600 panels. Totally you would have (first + last) times (n/2) which is (30+600) x (20/2) = 6300.

6. (20 points) The number of electric cars in a certain country grows exponentially at the rate of 20% per year. If there are 10,000 this year, find the equation for the number of electric cars t years from now.

Answer: Since it is growing continuously you get $10000(e^{0.2\,t})$.

If you wrote 10000(1.2^t) that is also fine.