

Howard University Math Department

Instructions:

PLEASE PROVIDE STEP BY STEP EXPLANATIONS

WRITING ONLY ANSWERS WILL NOT GET FULL CREDIT

Time Limit 30 minutes

Please read the questions carefully before answering

1. (10 points) Find the pattern by comparing the expressions below and write the n -th derivative product formula for $\frac{d^n}{dx^n}(fg)$ also written as $(fg)^{(n)}$. On the LHS you have expansions of $(x+y)^n$ using binomial formula. On the RHS you have the higher derivatives of the product of two functions f and g .

Note: $f^{(0)} = f$; $f^{(1)} = f' = df/dx$.

$$(x+y)^2 = x^2y^0 + 2xy + x^0y^2 ; \quad \frac{d^2}{dx^2}(fg) = (fg)^{(2)} = \frac{d}{dx}(f'g + fg') = f^{(2)}g + 2f'g' + fg^{(2)}.$$

$$(x+y)^3 = x^3y^0 + 3x^2y + 3xy^2 + x^0y^3 ; \quad \frac{d^3}{dx^3}(fg) = (fg)^{(3)} = f^{(3)}g + 3f^{(2)}g' + 3f'g^{(2)} + fg^{(3)}.$$

2. (20 points) Prove the formula you got in (1) using the basic proof by induction method,